

The literature has identified some problematic ways in which learners can reason about integers.

1. Right-to-left

This reasoning is characterised by learners operating from the right to the left in order to subtract a smaller number from a larger number.

Example:

$$4 - 9 = 5.$$

Can't subtract 9 from 4, so you treat as  $9 - 5 = 4$

2. Hidden brackets

The learners insert hidden brackets around pairs of numbers in order to 'do first'. Sometimes the brackets can be explicit as well.

Examples:

$$\begin{aligned} & 3 + 5 - 4 + 7 \\ & = (3 + 5) - (4 + 7) \\ & = 8 - 11 \\ & = -3 \end{aligned}$$

$$\begin{aligned} & 5 - 3 + 7 - 4 \\ & = 2 + 3 \\ & = 5 \end{aligned}$$

Does not result in an error

3. Sign rules

The learner uses sign rules from multiplication, such as 'minus and a minus give a plus' or 'a minus and a plus are a minus', when operating with integers. Because of the duality of the minus symbol being both a sign and an operation, for some learners, a subtraction symbol can be taken as a 'minus' and used in the sign rules.

Examples:

$$- 3 - 7 = 10$$

$$- 3 - (- 7) = -10$$

"Minus  $[- 3 - (- 7)]$  with minus  $[- 3 - (- 7)]$  give plus, and plus with a minus  $[- 3 - (- 7)]$  is minus. 3 AND 7 are 10."

Some learners may have been exposed to rules such as "if the signs are different, subtract and take the sign of the bigger". But these can be incorrectly recalled or applied.

Examples:

$$2 + (- 4) = - 6$$

"Add and take the sign of the bigger (-4)"

Or learners could argue  $2 > -4$ , so the answer is

4. Too many signs

When there are adjacent symbols, i.e. an operation followed by a sign as in  $8 - (- 5)$ , then one of the signs is not required.

Example:

$$\begin{aligned} & 8 - (- 5) \\ & = 8 - 5 \\ & = 3 \end{aligned}$$

5. Taking the following sign

This is more prevalent in algebra, but is possible if learners are 'rearranging' an expression to get 'nicer' numbers together.

Example:

$$\begin{aligned} & 7 - 8 + 3 - 2 \\ & = 7 - 8 + 2 - 3 \\ & = 7 - 10 - 3 \\ & = 7 - 7 \\ & = 0 \end{aligned}$$

+ attached to  
8, move '2'.

6. Brackets – Multiplication or addition/subtraction

These errors were not noted in literature but were seen in the integer task in Theme 1 earlier this year. Brackets can be interpreted as always indicating multiply.

Example:

$$\begin{aligned} & (-7) - 4 \\ & = 28 \end{aligned}$$

And then at other times, the reverse is true. When the notation requires multiplication, the brackets are dropped, possibly because 'there is nothing to do'.

Example:

$$\begin{aligned} & (-7)(-4) \\ & = -7 - 4 \\ & = -11 \end{aligned}$$