MISGSA 2021:
Covid-19 Mask Design

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Masks

The aim is to determine design principles for masks in a Covid-19 context for the wearer and victim; we want the best mask consistent with personal comfort.

The questions we asked were:

- What permeability (fabric thickness, cloth type and ,weave) and mask fitting parameters (size, shape, support) will be 'best'? and
- What are the implications in terms of droplet transfer/ Covid-19 spread.

We examined available literature in the area so we could make informed modelling choices.

Flow Behaviour

Observations:

- The air stream from the nose or mouth will either pass through the mask or leak out the sides of the mask.
- The volume of space between the mask and face must be sufficient and/or the mask flexible enough to allow for comfortable air exchange due to breathing.

Question: If δ is the average thickness of the gap around the mask edge (perimeter L_0, area A_0), and k is the (global) permeability (so that flux/area through the mask fabric $q = -kp_x$), determine the (leakage flux)/(mask through flux) ratio as a function of relevant parameters under sneeze, cough or normal conditions.

The Dbouk Reference: ‘Respiratory droplets and face masks’

This investigation (involved detailed computations of the flow field together with droplet dispersal, breakup, evaporation,....., filter action) is based on available data and ‘state of the art’ dimensional modelling. (images given)
Observations:

- In the absence of a mask intermediate sized $< 5\ \mu$ m sneeze droplets reach a distance of about 70 cm diameter Aerosols (¡$< 5\mu$ m) remain in the air. (relative importance?)
- Mask fitting important. Mask efficiency decreases during a cough cycle.
- Even with professional masks (N95) droplets leak around the sides of the mask and pass through the mask.
- Droplet distribution, flux levels, length and time scales, are provided (1.2 secs for a cough) and dimensionless groups identified.

Data

- Spacing between face and mask 4 mm to 1.4 cm
- Droplet diameter range 1 μm to 300 μm. Average diameter 80 μm; distribution given. Typically 1000 droplets released. Mass of siliva 7.7 mg from the mouth (single incident?). From single sneeze 0.12 mg.
- Cough duration (single cough) 0.12 sec. Typically 10 coughs in a cycle; cycle time 0.4 secs. Cycling significantly effects fluids and droplets interactions.
- Nose air velocity 0.4 m/sec up to 5.0 m/sec. Reynolds number 4400 (based on mouth hydraulic diameter).
- Maximum pore size in filter 47 to 146 μ m. Porosity model see (3). Mask thickness 2 mm.
- Filter efficiency see (6)

Suggested further research? (Dbouk)

- Droplet breakup, coalescence, capture in the filter (pore microstructure/droplet size)
- Cough dynamics (medical conditions)
- The source of droplets: Saliva droplet composition (surface energy, jet stream strength....)
- More advanced filters

Conceptual Framework for a Simple Flow Model

- Mask types: many: some solid, some extremely flexible (folds etc), some with large enclosed space. . . . Our primary concern is with the flow behaviour, droplet dispersal, under sneeze/cough conditions.
- Our aim is to produce the model that incorporates the **major features** of the problem. A complex/detailed model, even if available, would not (generally) provide useful 'design principles'. One would anticipate/hope that the models we develop would be calibrated using a simple experimental setup.

The Flow Equations

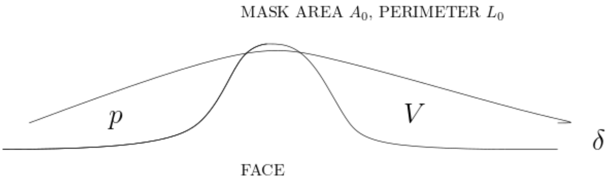

Figure 1: Mask flow

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Leakage:
Masks fit reasonably snugly on the face but there will be a space (volume V) between the face and the mask and there will be an average separation distance δ around the mask edge. The effect of a cough or a sneeze (or indeed simply an out-breath) will be to cause an increase in air pressure p (above atmosphere) within this space. This in turn will cause an increase in volume V within this

Steady state Bernoulli Model

Steady state Bernoulli's equation gives (many approximations here)

$$\boxed{p + 1/2\rho v^2 = constant}$$

Thus we get an expression for the velocity of air particles escaping from the enclosed space through the gap as a result of internal pressure p as

$$v = \sqrt{2p/\rho} \tag{1}$$

with the associated total flux

$$Q_s = \alpha\sqrt{2p/\rho}(L_0\delta), \tag{2}$$

eq:

where $\alpha \approx 0.6$ is a fitting parameter which takes into account the inadequacies of the model.

Through-flow; membrane equation

We treat the mask material as a membrane with flow-through behaviour described by

$$\boxed{q = kp}; \tag{3}$$

q the flux per unit area out the mask, p the pressure difference across the mask face, and the bulk permeability k depends on the mask filtering material (area A_0).
The total flux through the mask face is thus

$$Q_m = kpA_0 \tag{4}$$

eq:

Sneeze/cough input

Mass conservation in the mask space requires

$$\boxed{\frac{dV}{dt} = Q_{in} - [Q_s + Q_m]}; \qquad (5)$$

eq:

where $Q_{in}(t)$ is the (prescribed) total volume flux from the cough/sneeze (m^3/sec) and you'll recall $V = V(t)$ is the volume of air under the mask.
Here $p = p(t)$, and Q_m, Q_s are functions of $p(t)$.
If we assume an equation of state $V(p)$ connecting p and V, see later then:

Mask space dynamics

Equation (5) eq:Vteqn becomes:

$$\boxed{[\frac{dV}{dp}]\frac{dp}{dt} = Q_{in}(t) - [Q_s(p) + Q_m(p)]}, \qquad (6)$$

eq:

where we've explicitly noted the dependence mask fluxes on the pressure $p(t)$ within the mask air gap.
This is an ordinary differential equation determining $p(t)$ which can be solved for particular flux inputs $Q_{in}(t)$ (nose or mouth).
The 'mask response' factor $[\frac{dV}{dp}]$ depends on the state equation for the mask; cases below.

Scaling

We introduce scales so as to reduce the equation to its simplest form and identify the important dimensionless groups. We write

$$Q_{in} = \bar{Q}Q'_{in}(t'), \bar{Q}Q'_{m}(t'), Q_s = \bar{Q}Q'_s(t'),\ t = t_0 t', p = p_0 p', v = V_0 V', \tag{7}$$

where $\bar{Q}$ is a typical air flux (m^3/sec) from the nose or mouth, V_0 a typical mask space volume, and we choose (t_0, p_0) to reduce the equation to its simplest dimensionless form:

$$[\frac{dV'}{dp'}]\frac{dp'}{dt'} = Q'_{in}1 - [\xi_s\sqrt{p'} + p']; \tag{8}$$

eq:

$$\boxed{\xi_s = \frac{\alpha}{k}\sqrt{\frac{2}{(\rho p)}}[\frac{L_0\delta}{A_0}]}. \tag{9}$$

The dimensionless flux ratio parameter ξ_s provides a measure for the ratio of the side (leakage) flux to the mask (face) flux and can be thought as a *fitting parameter.*

Flux ratio ξ_s: Different quality masks

Dropping primes we get

$$\boxed{[\frac{dV}{dp}]\frac{dp}{dt} = Q_{in} - [\xi_s\sqrt{p} + p]}, \qquad (10)$$

eq:

If ξ_s is large then the mask is relatively 'leaky', that is not well sealed and/or with dense mask material, whereas small values correspond to a well sealed, small permeability k mask.

The *mask design function* $[\frac{dV}{dp}]$ describes the mask behaviour under inflation; for example cloth masks with folds first inflate easily, and then with difficulty, see later.

Note that with our model just two factors, ξ_s and $[\frac{dV}{dp}]$, are needed to characterise the air exchange behaviour of masks.

We first look at the simple example of a fixed geometry mask.

Steady State or rigid mask behaviour

If the mask is rigid then no change in the mask volume space V occurs during a sneeze or cough. In this case $\frac{dV}{dt} = 0$, so (dropping primes)

$$Q_{in} = \xi_s \sqrt{p} + p; \tag{11}$$

(a quadratic in $\sqrt{p}$) which determines the pressure p due to any prescribed flux input $Q_{in}(t)$ (a sneeze). The associated leakage and membrane fluxes can be recovered using (2,4).

Note that if p is small in (11) then the $\sqrt{p}$ dominates but the linear term takes over as p increases. Thus the primary loss is through leakage initially and later as p increases throughflow takes over.

Pressure buildup as Q_{in} increases

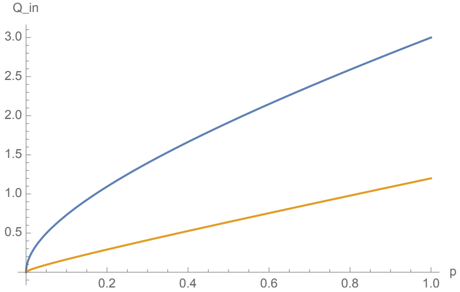

Figure 2: Behaviour of solid masks with different ξ_s's under inflation: top curve $\xi_s = 2$ (a leaky mask), bottom curve $\xi_s = 0.2$ (a well fitted mask)

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Note that the leakage flux initially increases very rapidly as p increases.

Flux apportionment: steady state

As p increases the efficiency of the mask increases because of the enhanced throughflow.

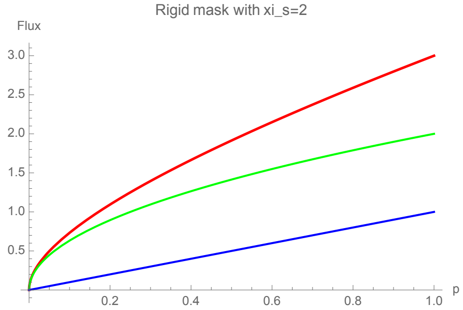
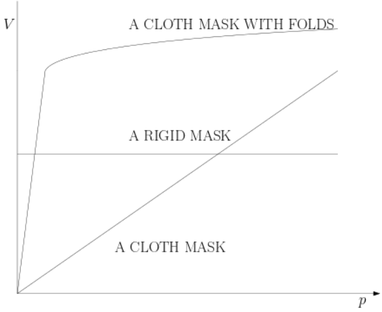


Figure 3: Flux apportionment: Flux input Q_{in} (red), leakage flux component Q_s (green), throughflux Q_m (blue)

State equation $V(p)$ for nonrigid face masks.

Simple cloth masks expand uniformly under increasing pressure, whereas cloth masks with folds expand rapidly (with folds unfolding) with increasing pressure until the the cloth is 'fully stretched, then the expansion rate is very slow. The associated state diagrams are displayed in the figure, the rigid mask is also included.



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Response curves for Impulsive Sneeze/Cough flows

We model a sneeze as a fixed flux input over a small time interval (0.1 secs), and determine the response for different mask types.
We thus assume that the strength of the air stream isn't effected by the presence of the mask.
We compare results for $\xi_s = 2$, 'a leaky mask', and $\xi_s = 0.2$, a 'well fitted mask'.
We then repeat the exercise for masks of various types as defined by the state equation $V(p)$: Case 2; a rigid mask, Case2 a cloth mask, Case 3 a cloth mask with folds.
If time permits we will then consider a train of coughs, and normal breathing.

A cloth mask: Case 2, $\xi_s = 2$

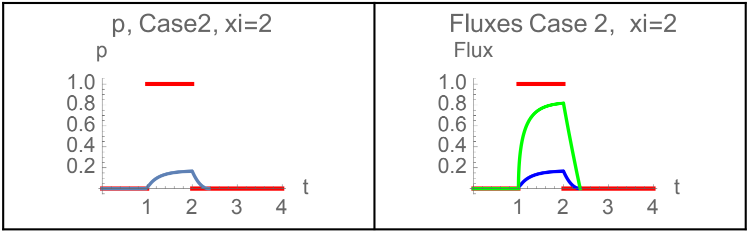

Figure 4: Case 2:Cloth mask (poorly fitted): Left: $Q_{in}(t)$ (red), $p(t)$ (blue) Right: leak flux Q_s (green), membrane flux Q_m (blue)

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Note that the mask space pressure increases rapidly initially and after the sneeze finishes there is an exponential reduction to zero (atmospheric pressure). As expected with this poorly fitted mask much of the flux from the sneeze leaks out the sides.

A cloth mask: Case 2, $\xi_s = 0.2$

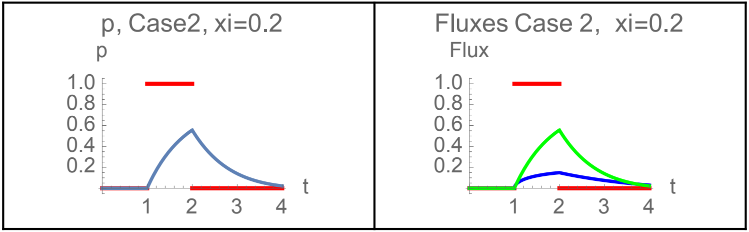

Figure 5: Case 2:Cloth mask (well fitted): Left: $Q_{in}(t)$ (red), $p(t)$ (blue)
Right: leak flux Q_s (green), membrane flux Q_m (blue)

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Note the increased mask pressure and throughflux and reduced leakage. Also note that the mask remains inflated longer.

Model Usefulness?

The simplicity of the model is compelling in that few simple experiments are required (simply determine $V(p)$ using a plastic mode and estimate ξ) to calibrate masks. Any further 'sophistication' in either the fluids description or the mask response to forcing would require **much** more and hard to collect and less universally useful data.

Assumptions Made/possible corrections

- Tbe steady state Bernoulli equation assumes:
  1. The response time of the mask, and the flow within the mask space are small (for a sneeze?)
  2. Viscous effects are relatively small compared with inertia effects (what if the mask space is thin?)
  3. A correction factor α works here
- A equation of state can be used to determine the mask with support response. When the mask is deflating the a state equation description is probably not possible?
- Other?

Summary: Major modelling Challenge

- Complex problem Simple practical solution needed
- Simple mask flow model set up but with suspect assumptions
- Modifications/refinements needed if basic assumptions are OK
- More theoretical and experimental work needed to assess the model's usefulness.
- This crude fluid flow model may not be adequate for determining droplet dispersal, since actual fluid paths are not determined with this model.