Decision Support Tool for Optimal Beer Blending

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Presentation Overview

Introduction

- There is a decrease in the consumption of traditional beers, as a result of consumers seeking more adventurous tastes.
- Brewing companies are forced to propose more varieties of beers to suite various markets.
- Beer varieties are uniquely blended to obtain different types of beer blends with each satisfying different attributes at specific levels.

Introduction

- The problem comes from a North American based brewing company, which is well established with concept of beer blending.
- The company is in the process of having more varieties of beers on the market, they are considering a wider range of raw materials with wider range of attributes.
- Blending has become a complex task since they have to process large raw material and attributes to produce quality beer blends at lowest cost.

Company Aim

Develop and solve a blending model that is able to :

- Determine the closest match of beer blends at the lowest cost.
- Allow the user to analyse the trade-off between the quality of the blends and the cost of the raw materials.

Problem Formulation

Specific criteria :

- Minimize total production cost per week.
- Adhere as closely as possible to qualitative characteristics of existing blends.

What is an optimal solution ?

- There may be an infinite number of solutions which may be said to be *optimal*.
- Each *optima* represents a trade-off (in this case cost vs quality).
- The client may have preferences or priorities that will guide or determine the choice of the final solution.

Target Blend Characteristics

$$B = \begin{bmatrix} \textbf{Blend 1} & \dots & \dots & \dots & \textbf{Blend N} \\ b_{11} & \dots & \dots & \dots & b_{1N} \\ \vdots & \ddots & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ b_{M1} & \dots & \dots & \dots & b_{MN} \end{bmatrix}$$

- Number of blends : N
- Number of attributes : P
- Variable indexing blends : j
- Variable indexing attributes : k

Raw Material Characteristics

$$R = \begin{bmatrix} \textbf{Raw Materials} & \textbf{Attribute 1} & \dots & \dots & \textbf{Attribute } P \\ RM1 & r_{11} & \dots & \dots & r_{1P} \\ \vdots & \ddots & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ RMM & r_{M1} & \dots & \dots & r_{MP} \end{bmatrix}$$

- Number of raw materials : M
- Number of attributes : P
- Variable indexing raw materials : i

Supply, Demand and Cost Constraints

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ \vdots \\ d_N \end{bmatrix}$$

TABLE – Demand for each blend j

$$\mathbf{s} = \begin{bmatrix} s_1 \\ \vdots \\ \vdots \\ s_M \end{bmatrix}$$

TABLE – Supply for each raw material i

$$\mathbf{c} = \begin{bmatrix} c_1 \\ \vdots \\ \vdots \\ c_M \end{bmatrix}$$

TABLE – Cost of each raw material i per kg/week

Variables : Blend Recipes

$$X = \begin{bmatrix} \textbf{Blend 1} & \dots & \dots & \dots & \textbf{Blend N} \\ x_{11} & \cdots & \cdots & \cdots & x_{1N} \\ \vdots & \ddots & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ x_{M1} & \cdots & \cdots & \cdots & x_{MN} \end{bmatrix}$$

- where x_{ij} is the amount in kilograms of raw materials i in blend j

Choice of models

We explore two approaches to optimizing over multiple objectives :

- Bounded-ϵ.
- Weighted sum.

Bounded-ϵ

- Objective function :
  1. Optimize over a single objective : Total cost (C).
  2. Allow the other objectives to vary within an acceptable range and add to list of constraints below.
- Constraints :
  1. Limited supply of materials
  2. Minimum demand from distributors.
  3. Accuracy of blend (constrained ϵ).
- Assumptions :
  1. Each blend characteristic is of equal importance.
  2. Difference in quality can be tasted when $\epsilon > 1.0$.

Bounded-ϵ

- Objective function :

$$C = \sum_{j=1}^{N} \sum_{i=1}^{M} c_i x_{ij}$$

- Subject to :

$$\sum_{j=1}^{N} x_{ij} \leq s_i, \forall i \in [1, M] \text{ - Supply of materials}$$

$$\sum_{i=1}^{M} x_{ij} \geq d_j, \forall j \in [1, N] \text{ - Demand of blend}$$

$$\frac{\sum_{i=1}^{M} r_{ik} x_{ij}}{\sum_{i=1}^{M} x_{ij}} = b_{jk}, \forall j \in [1, N], \forall k \in [1, P] \text{ - Accuracy of blend}$$

Bounded-ϵ

- Objective function :

$$C = \sum_{j=1}^{N} \sum_{i=1}^{M} c_i x_{ij}$$

- Subject to :

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$$\sum_{i=1}^{M} x_{ij} \geq d_j, \forall j \in [1, N] \text{ - Demand of blend}$$

$$b_{jk} - \epsilon_{jk} \leq \frac{\sum_{i=1}^{M} r_{ik} x_{ij}}{|x_j|} \leq b_{jk} + \epsilon_{jk}, \forall j \in [1, N], \forall k \in [1, P]$$

- Accuracy of blend ,$\epsilon_{jk} \in [0, 1]$

Bounded-ϵ

- Objective function :

$$C = \sum_{j=1}^{N} \sum_{i=1}^{M} c_i x_{ij}$$

- Subject to :

$$\sum_{j=1}^{N} x_{ij} \leq s_i, \forall i \in [1, M] \text{ - Supply of materials}$$

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$$\frac{\sum_{i=1}^{M} r_{ik} x_{ij}}{|x_j|} \geq b_{jk} - \epsilon_{jk}, \forall j \in [1, N], \forall k \in [1, P] \ , \epsilon_{jk} \in [0, 1]$$

$$\frac{\sum_{i=1}^{M} r_{ik} x_{ij}}{|x_j|} \leq b_{jk} + \epsilon_{jk}, \forall j \in [1, N], \forall k \in [1, P] \ , \epsilon_{jk} \in [0, 1]$$

Bounded-ϵ

- Objective function :

$$C = \sum_{j=1}^{N} \sum_{i=1}^{M} c_i x_{ij}$$

- Subject to :

$$\sum_{j=1}^{N} x_{ij} \leq s_i, \forall i \in [1, M] \text{ - Supply of materials}$$

$$\sum_{i=1}^{M} x_{ij} \geq d_j, \forall j \in [1, N] \text{ - Demand of blend}$$

$$\sum_{i=1}^{M} (r_{ik} - b_{jk} + \epsilon_{jk}) x_{ij} \geq 0, \forall j \in [1, N], \forall k \in [1, P] \ , \epsilon_{jk} \in [0, 1]$$

$$\sum_{i=1}^{M} (r_{ik} - b_{jk} - \epsilon_{jk}) x_{ij} \leq 0, \forall j \in [1, N], \forall k \in [1, P] \ , \epsilon_{jk} \in [0, 1]$$

Weighted Sum

- Objective function :
  1. Consider multiple objectives simultaneously.
  2. Introduce an additional set of auxiliary variables that represent closeness to desired qualitative characteristics
  3. Apply a weighted sum to all the objectives to obtain a single metric
  4. Optimize over this summary descriptor
- Constraints :
  1. Limited supply of materials
  2. Minimum demand from distributors
  3. Auxiliary variables constrained by target blend characteristics
- Assumptions :
  1. Each blend characteristic is of equal importance
  2. Difference in quality can be tasted when $\epsilon > 1.0$

Weighted Sum

- Objective function :

$$c_1(x) = min \sum_{j=1}^{N} \sum_{i=1}^{M} c_i x_{ij}$$

$$c_2(x) = min y_{jk}, \forall j \in [1, N], \forall k \in [1, P]$$

- Subject to :

$$\sum_{j=1}^{N} x_{ij} \leq s_i, \forall i \in [1, M] \text{ - Supply of materials}$$

$$\sum_{i=1}^{M} x_{ij} \geq d_j, \forall j \in [1, N] \text{ - Demand of blend}$$

$$y_{jk} \geq |\sum_{i=1}^{M} (r_{ik} - b_{jk}) x_{ij}|, \forall j \in [1, N], \forall k \in [1, P]$$

Weighted Sum

- Objective function :

$$C = \lambda_1 c_1(x) + \lambda_2 c_2(x), \text{where} \sum_i \lambda_i = 1$$

- Subject to :

$$\sum_{j=1}^{N} x_{ij} \leq s_i, \forall i \in [1, M] \text{ - Supply of materials}$$

$$\sum_{i=1}^{M} x_{ij} \geq d_j, \forall j \in [1, N] \text{ - Demand of blend}$$

$$y_{jk} \geq \sum_{i=1}^{M} (r_{ik} - b_{jk}) x_{ij}, \forall j \in [1, N], \forall k \in [1, P]$$

$$y_{jk} \leq -\sum_{i=1}^{M} (r_{ik} - b_{jk}) x_{ij}, \forall j \in [1, N], \forall k \in [1, P]$$

Weighted Sum

Two important questions :

- How did we set the weights λ_i ?
  - We viewed the weights as a trade-off between cost and quality.
  - However, often the solution obtained does not reflect the preferences expressed in the choice of weights.
- Should we scale $c1(x)$ and $c2(x)$?
  - Objective functions may be measured in different units and may have different orders of magnitude.
  - In this case, y_{jk} is a proxy for blend characteristics and we hope $y_{jk} <= 1.0$, whereas production cost $= \$1,000,000$.
  - However, y_{jk} may assume very large values $(100,000 - 1,000,000)$ during numerical solution, not directly representative of blend closeness to target characteristic.
  - Some authors discourage scaling of objective functions when weights are used as trade-offs ($\sum_i \lambda_i = 1$) and our empirical results confirm this.

Numerical Results

Run experiments for :

- Bounded-ϵ :
  - several runs where $\epsilon \in [0.3, 1.0]$
  - No solutions found when $\epsilon < 0.3$
- Weighted sum :
  - several runs where $\lambda_1 \in [0.1, 0.9]$
  - $\lambda_2 = 1.0 - \lambda_1$

Results : Cost vs Quality

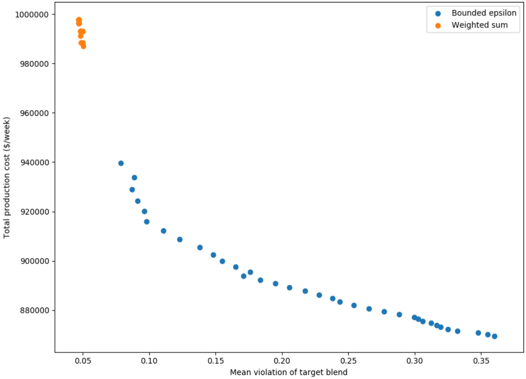

FIGURE – Trade-off between cost and adherence to target characteristics

Comparison : Mean violations

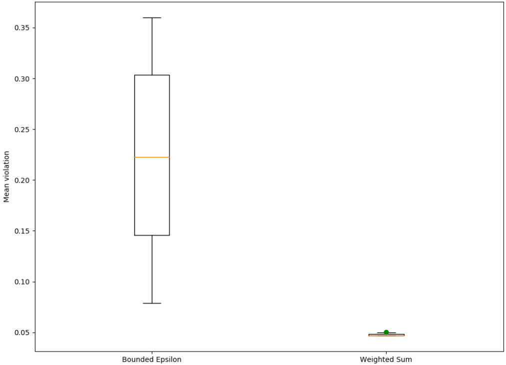

FIGURE – Distribution of mean violations of blend characteristics over all experiments

Comparison : Maximum violation

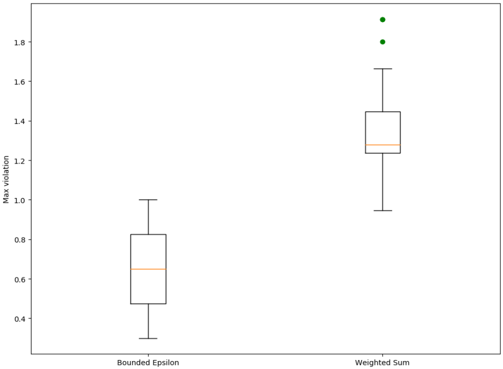

FIGURE – Distribution of maximum violation of blend characteristics over all experiments

Comparison : Total number of violations

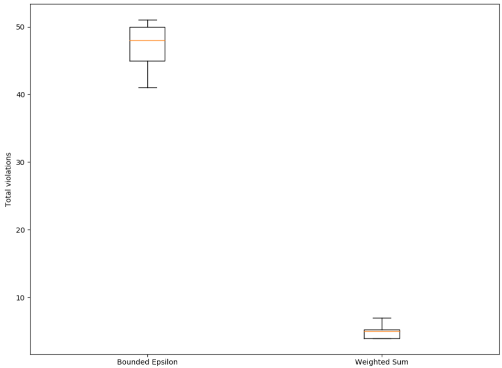

FIGURE – Distribution of number of violations blend characteristics over all experiments ($\delta \geq 0.05$)

Comparison : Typical Candidate Solutions

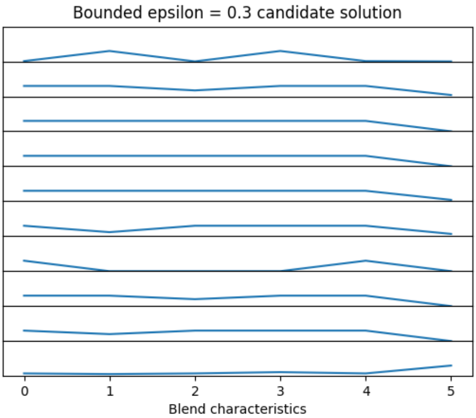

FIGURE – Candidate solution displays several mild deviations from the target blend characteristics

Comparison : Typical Candidate Solutions

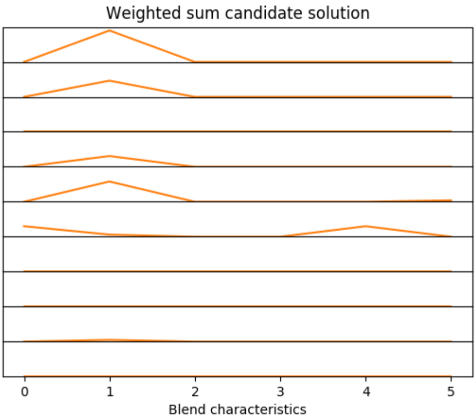

FIGURE – Candidate solution displays a few sharp deviations from the target blend characteristics with near perfect match elsewhere

Conclusion

- Solutions represent a trade-off between cost and quality, where the client's preferences will guide the final choice of model.
- When searching through a possibly infinite set of candidate solutions, different approaches may yield solution sets with special characteristics.
- Different approaches may fill in parts of the solution space which would be inaccessible if one follows only one method.
- Despite a large range of weight preferences, the weighted sum method favours solutions that prioritize quality over cost.

Further work

- For Bounded-ϵ : introduce ϵ_{jk} for expert to set individual preferences on blend characteristics.
- For Weighted Sum : introduce w_{jk} weights for expert to set individual weights on y_{jk}.
- These more general weights will also allow for Monte Carlo Search in the solution space.
- Explore further approaches like Genetic Algorithm Search and other Meta-heuristic techniques.

References