TRANSFORMING SOUTH AFRICAN SUGARCANE FACTORIES TO BIOREFINERIES

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Abstract

The study group was asked to consider methods for determining the optimal use of sugarcane grown in the South African industry with particular regard to diversification of the product to increase profitability. This proved to be a very difficult problem given the large number of possible products and costs. The group was able to summarize the techniques that might be used by considering two simplified model sub-problems and considering optimal outcomes for each. The overall "solution" could be obtained by applying these techniques across all processes of the refineries.

1 Introduction

South African sugar refineries produce sugar for both the domestic and international markets. The processes involved in making sugar produce a number of by-products, and great efficiencies are to be gained by recycling. In addition, some of the by-products can be used to generate energy which can be used to power the process, making the factory self-sufficient in energy. While its major product is sugar, there are a number of other products that could be produced and sold. In particular, sugar

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cane is an ideal renewable resource for biofuels as it is an extremely efficient converter of sunlight and carbon dioxide. In fact, there is a complicated combination of choices made by the management that may increase both profitability and efficiency of the whole operation.

The question for the study group was "Can profitability be increased by alternative use of by-products to make ethanol, bio-polymers or generate power, ...?" This decision has to be made in the context of providing sufficient sugar products to the local market. The problem is therefore to consider all of the possible outcomes, their cost structures, capital investment required and marketability to find the optimal long term strategy for the local industry.

This decision making process can therefore be considered as a large nonlinear, dynamic optimization problem. Since it was unlikely that the full problem could be formulated and solved in the time available at the MISG, the group decided to create a simplified model including several different processes and outcomes and hence create a framework for the much larger model that may be worked on as a follow-up. It was therefore decided to consider two smaller problems; a single process requiring optimization within a single process and the various possible uses of a single by-product in the other, thus deriving the appropriate mathematical processes for decision making. These techniques can be applied in principle to other processes, groups of processes or the whole operation. There are some papers which do similar things for different parts of the refinement process, such as [4]. Much more detail of the sugar refining process can be found in books such as [1] and [2].

2 Model Problem 1

A number of processes within the plant consist of various options that may be taken by managers. One such problem involves so-called "Imbibition", or the addition of water to sugarcane to extract the sucrose. The more water added per ton, the more sucrose that can be obtained. Suppose that the amount of sucrose extracted per ton of sugarcane can be written as

$$E = 1 - e^{-5.72I} \qquad (1)$$

where I is the imbibition ratio or ratio of mass of water to mass of sugarcane and E is the proportion of the total amount of sucrose available per ton of sugarcane (often around 15% [3]) . The more water that is added, the more of the sucrose that can be extracted. This is not an exact formula but is typical of the behaviour of this process, as discussed in [5, 6]. This trial form was provided by the industry representatives.

However, the process requires steam for the extraction. Burning 100 tons of bagasse (a bi-product of the process) can produce 55 tons of steam, but the total amount of steam required is estimated to be $S = 50 + 25I$ tons per 100 tons of cane. Therefore, if more than 55 tons is required the steam must be generated by purchasing and burning coal at a cost of around C per ton, with each ton producing 10 tons of steam. While most of the quantities quoted are fixed by the process, the price of coal is variable and hence we leave it undefined for now. Since (relatively) the cost of burning bagasse is trivial, the profit function is only moderated by coal if $I > 0.2$, since then more than 55 tons of steam are required. Therefore, if the sale price of sucrose is P_S per ton we can write the profit V per 100 tons as

$$V(I, C, P_S) = \begin{cases} 15P_S(1 - e^{-5.72I}), & I \leq 0.2 \\ 15P_S(1 - e^{-5.72I}) - \frac{C}{10}(25I - 5), & I > 0.2. \end{cases} \tag{2}$$

To choose the imbibition rate, I, to maximize the profit, we note that if C and P_S are fixed,

$$\frac{dV}{dI} = 85.8P_S \; e^{-5.72I} - 2.5C = 0$$

which implies that

$$I_{\text{opt}} = \frac{-1}{5.72} \log \left[\frac{2.5C}{85.8 \; P_S} \right] \tag{3}$$

is the optimal value of I. Using a typical coal price of $C =$ R 1500/ton and sucrose sale price $P_S =$ R 3400/ton, then $I_{\text{opt}} = 0.7612$ and the profit is $V_{\max} =$ R 482/ton.

It is also of interest to consider how this may change if the price of coal or sale price of sucrose change. At least locally, this can be determined by taking partial derivatives of V with respect to C and P_S to see their influence on V. Then we have

$$\begin{aligned} \frac{\partial V}{\partial C} &= -2.5I - 0.5 \; , \\ \frac{\partial V}{\partial P_S} &= 15\big(1 - e^{-5.72I}\big) \; , \end{aligned}$$

which means that close to the optimal value of I, a 1% increase in cost of coal will reduce the profitability by around 0.8%, while an increase in sale price of sucrose of 1% will increase profitability by about 1.08%. This suggests that unless there is quite a large change in price of sucrose or cost of coal then modifying the value of “Imbibition” will only marginally improve the outcome. However, it may be worth computing the outcome over a period of time and monitoring the profit/loss computed by either changing I or leaving it the same as prices fluctuate.

3 Model Problem 2

In this problem we have to deal with a choice between several products to optimize the profit from a single input. Sugar cane consists of a range of components including glucose, fibre, sucrose, fructose, cellulose and starch along with 70% water. In our

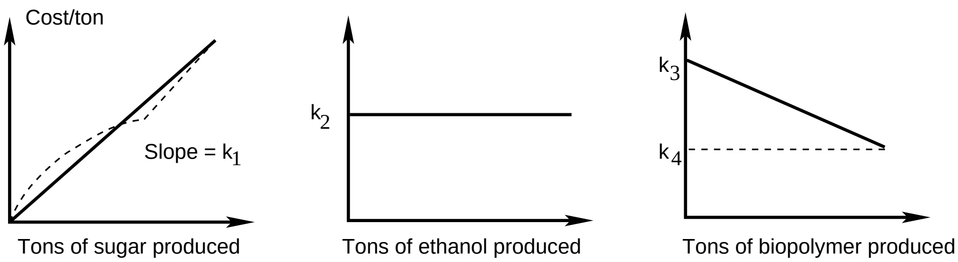

Figure 1: Cost functions for sugar, ethanol and biopolmers as approximated in the model

model problem we considered only the "juice" derived from the extraction process. This juice can be sub-divided in the following ways, giving three possible products with different values, costs, etc.

$$\text{Sugarcane} \Rightarrow \text{Extraction} \Rightarrow \text{Juice} \rightarrow \begin{cases} \text{Sugar} \\ \text{Ethanol} \\ \text{Biopolymer} \end{cases} \rightarrow \ldots \text{Market} \tag{4}$$

The goal of the group was to formulate the problem for the uses of this juice and then consider optimization of profit given different scenarios. Thus the problem can be thought of as to find the combination of proportions of each product that will maximize the profit. The issue of time scales is important here, since some processes involve a good deal of capital expenditure to set up (for example, bio-refinery), while others can be set up quickly. Short term profit may not be optimal in the long term. However, we begin with consideration of a simple model.

In this model, we simply set up three possible choices for use of the juice and give them reasonable cost and sale price functions. Given these we can then find the optimal outcome. Therefore, the value, V, of each product including costs and sale price can be written as

$$V_k = A_k \left(P_k - C(A_k) \right), \quad k = 1, 2, 3 ,$$

where A_k =amount (proportion) of product k, P_k=price of product k, $C(A_k)$=cost function for product k which will clearly depend on A_k. Each product will have different cost functions. In this work we will assume that the price is constant, although it is possible to relax this assumption. The three products we will consider are sugar ($k = 1$), ethanol ($k = 2$) and biopolymer ($k = 3$). The amounts, $A_k, k = 1, 2, 3$, are scaled to be a proportion of the total amount of juice, so we can say that $A_1 + A_2 + A_3 = 1$. Using information provided by the sugar industry, reasonable functions for the cost structure of these items can be seen in Figure 1.

In the case of sugar, a linear growth in cost (with rate k_1) with the amount of sugar produced, A_1, was assumed and the price was assumed to be P_1, giving

$$V_1 = A_1(P_1 - k_1 A_1) \ . \tag{5}$$

This is an approximation to the real situation (see the dashed line in Figure 1 (a)). While it may seem strange to have this increase in cost per yield, it is related to the cost of extracting more sugar from a fixed crop, as in Model 1 above. Clearly the cost function is more complicated in reality, but we have used this as an example only.

Ethanol was assumed to have a constant cost per ton, k_2, independent of the amount produced, A_2, so that

$$V_2 = A_2(P_2 - k_2) \ , \tag{6}$$

where P_2 is the price of ethanol.

Our third product was more complicated because there is a fixed set-up cost, k_3, (independent of amount) but then decreasing costs (rate k_4) as more is produced, giving a value function

$$V_3 = A_3(P_3 - (k_3 - k_4)A_3 - k_3) \ , \tag{7}$$

where A_3 is the amount of biopolymer and P_3 is the sale price. Note that all quantities are per ton of the original juice.

3.1 Mathematical formulation

Given these cost functions, we can set up a simple optimization problem by computing the maximum value per ton of the total value of the three items under consideration, so that the problem becomes;

$$\begin{array}{ll} \max & V = V_1 + V_2 + V_3 \\ \text{s.t} & \quad A_1 + A_2 + A_3 = 1 \\ & \quad A_1 \geq 0; A_2 \geq 0; A_3 \geq 0 \end{array} \tag{8}$$

where V_1, V_2 and V_3 are given by equations (5),(6),(7), respectively.

Note that this can be reduced to a two-dimensional problem by using the conservation law $A_2 = 1 - A_1 - A_3$. This condition comes from the fact that we have a fixed amount of juice (which can be scaled to be one).

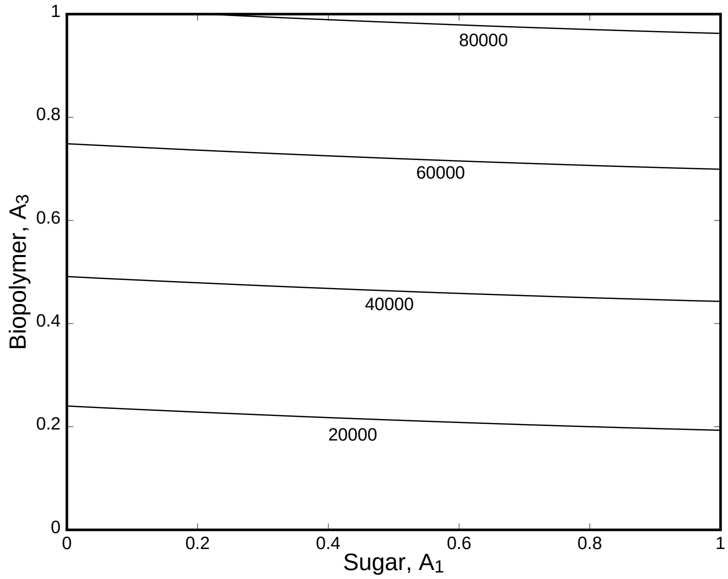

Figure 2: Contours of value given data in (12). The optimal feasible solution here is $A_1 = 0, A_2 = 0$ and $A_3 = 1 \Rightarrow V_{\max} = 79000$

3.2 Direct solution

We can solve this simple problem in the usual way. Writing the problem in terms of A_1 and A_3, we have

$$\begin{aligned} \max \quad V &= V_1 + V_2 + V_3 \\ &= A_1(P_1 - \beta) - k_1 A_1^2 + \beta + A_3(\alpha - \beta) - \gamma A_3^2 \\ \text{where} \quad \alpha &= P_3 - k_3, \quad \beta = P_2 - k_2, \quad \text{and} \quad \gamma = k_3 - k_4. \end{aligned} \tag{9}$$

Taking partial derivatives with respect to A_1 and A_3 gives stationary points at

$$A_1 = \frac{P_1 - \beta}{2k_1} \quad \text{and} \quad A_3 = \frac{\alpha - \beta}{2\gamma} \tag{10}$$

and it is easy to verify that these give a local maximum provided $k_3 > k_4$, when

$$V_{\max} = \frac{(P_1 - \beta)^2}{2k_1} + \frac{(\alpha - \beta)^2}{2\gamma} + \beta. \tag{11}$$

In these equations it is clear that the sale prices of the product need to be based on the amount produced from a ton of juice. Taking estimates from the sugar research institute, we chose;

$$\begin{aligned} k_1 &= \text{R } 1200/\text{ton}, \quad k_2 = \text{R } 850/\text{ton}, \\ k_3 &= \text{R } 17000/\text{ton}, \quad k_4 = \text{R } 13000/\text{ton}, \\ P_1 &= \text{R } 5400, \quad P_2 = \text{R } 1250, \quad P_3 = \text{R } 100000. \end{aligned} \tag{12}$$

Contours of value given data in (12) are shown in Figure 2. Using these values, the optimal solution corresponds to $V_{\max} \approx 8.6 \times 10^5$ with $A_1 = 2.08$, $A_3 = 10.3$ and $A_2 = -11.38$ which is of course infeasible. The optimal feasible solution occurs when $A_3 = 1$ and $A_1 = A_2 = 0$ with $V_{\max} = 79,000$. Out of interest, solutions occur with $A_1 = 1, A_2 = A_3 = 0$, $V_{\max} = 4200$, and $A_2 = 1$, $A_1 = A_3 = 0$ with $V_{\max} = 400$. The reason for the dominance of product 3, the biopolymer, is the high price for it relative to the other products.

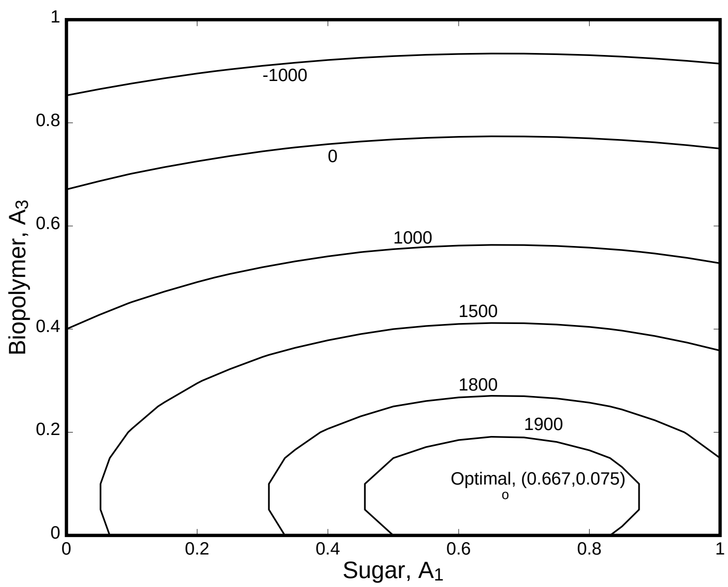

Figure 3: Contours of value given data in (13) The optimal solution of $A_1 \approx 0.667, A_2 \approx 0.259, A_3 \approx 0.075 \Rightarrow V_{\max} \approx 1956$ is indicated.

However, it is of interest to determine what would need to happen to create a situation in which a feasible optimal solution exists in which all products are utilised to some degree. One such solution is obtained by reducing the sale price of biopolymer to $P_3 = \text{R}19,000$, and increasing the sale price of the ethanol to $P_2 = \text{R}2,250$, in which case the optimal solution becomes

$$A_1 \approx 0.667, \quad A_2 \approx 0.259 \quad \text{and} \quad A_3 \approx 0.075 \quad \text{with} \quad V_{\max} \approx 1956. \tag{13}$$

The profit contours of this case are shown in Figure 3. This optimum compares with the 3 single-product options of

$$\begin{aligned} A_1 = 1, A_2 = A_3 = 0, &\quad V = 1800 \\ A_2 = 1, A_1 = A_3 = 0, &\quad V = 1850 \\ A_3 = 1, A_1 = A_2 = 0, &\quad V = -2000 \end{aligned}$$

In order to make the problem “interesting” we have had to reduce the price of biopolymer by 80%, and almost double the price of ethanol.

Therefore, in this problem the dominance of the price of A_3 in the case given by the original data provided renders the problem inadequate for the purposes of testing the general ideas to be employed. Consequently, we will use the modified values to examine the sensitivity of the problem.

3.3 Sensitivity analysis

Given that the different prices on the market are not fixed, we were interested in how changes in these may affect the value of V. To determine this we can examine the affect of each price individually on the optimal value. We can do this by considering the rate of change of V with respect to each of the prices, that is

$$\frac{\partial V}{\partial P_1} = A_1 \,, \qquad \frac{\partial V}{\partial P_2} = A_2 \,, \qquad \frac{\partial V}{\partial P_3} = A_3 \,,$$

which simply means that the overall value of the process with respect to each product is a linear function of the price. Perhaps a better indicator is to consider the effect of the cost on the amount produced, at the optimal value. In that case we take the optimal form of each of A_1, A_2 and A_3 and see how each is affected by the change in price of the other commodities. In that case;

$$\begin{aligned} &\frac{\partial A_1}{\partial P_1} = \frac{1}{2k_1}, \quad \frac{\partial A_1}{\partial P_2} = \frac{-1}{2k_1}, \quad \frac{\partial A_1}{\partial P_3} = 0, \\ &\frac{\partial A_2}{\partial P_1} = \frac{-1}{2k_1}, \quad \frac{\partial A_2}{\partial P_2} = \frac{1}{2k_1} + \frac{1}{2(k_3 - k_4)}, \quad \frac{\partial A_2}{\partial P_3} = \frac{-1}{2(k_3 - k_4)}, \\ &\frac{\partial A_3}{\partial P_1} = 0, \quad \frac{\partial A_3}{\partial P_2} = \frac{1}{2k_1} + \frac{-1}{2(k_3 - k_4)}, \quad \frac{\partial A_3}{\partial P_3} = \frac{1}{2(k_3 - k_4)} \,. \end{aligned}$$

This suggests that close to the optimum, a change in the price of sugar or ethanol has little effect on the other, the adjustment being done via a change in the amount of ethanol. Also, given that k_1 is quite a bit less than (k_3-k_4), the impact of changes in the price of P_1 seem to be greater than changes in P_3.

3.4 Remarks

This simple model containing three products encapsulates all of the basic optimization steps required to make decisions based on the data provided. Clearly, as the number of options increases, the problem becomes much more complicated and analytical solution far less likely. However, mathematical packages such as `Matlab`, `octave` and `scilab` have adequate tools for tackling such problems. The study group was able to replicate all of the above results using `Matlab`.

4 Stochastic simulation

In both of the problems above it is possible to consider the effect of volatility in the price of various products in the market and how it will alter the strategy of the sugar refinery. The study group decided to work with the second model only and perform stochastic simulations assuming a beta distribution of the the price of various components. The beta distribution was chosen because by varying the parameters a range of different shapes for the behaviour could be examined.

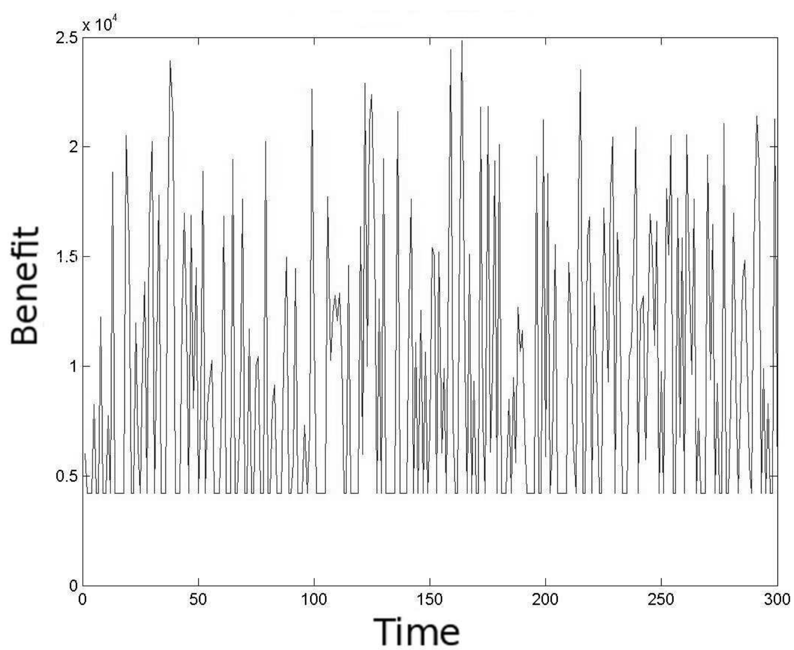

Figure 4: Typical evolution of the benefit over time for a single simulation.

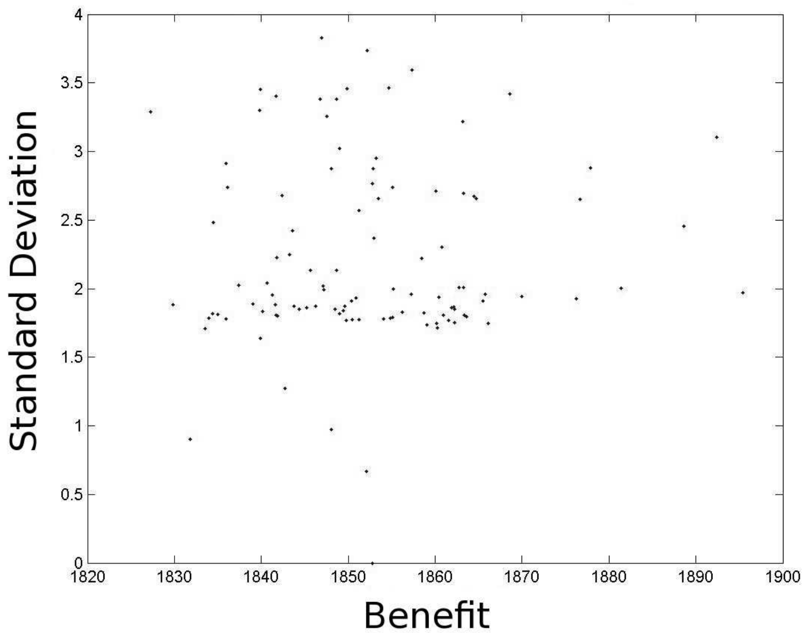

Figure 5: Benefit from a number of simulations with standard deviation of outcomes - a measure of risk

In this case the prices of all of the products were allowed to vary within a beta distribution. The profit over a 25 year period can then be calculated for a large number of simulations, thus providing a distribution of possible profit (benefit) if the price varies in this way. The simulations themself provide the range of possible outcomes, while the standard deviation of the outcomes provides a measure of "certainty" of profit given the appropriate strategy. Figure 4 shows the benefit function plotted against time (in months) over 25 years in a typical single simulation. There is quite a lot of scatter in the results, but this example is really a demonstration of what might be done.

Figure 5 shows a plot of benefit against standard deviation in the profit, to provide some estimate of the variability of the results. In this example, both product prices and the cost of the products was varied. This provides managers with a tool to evaluate the risk of each strategy.

5 Conclusion

The study group was asked to consider a very complicated problem involving a large number of unknowns. Instead of attempting to solve the full problem in the limited time available, two model problems were solved in order to demonstrate the techniques available and how risk assessment might be conducted. Sensitivity to price variation was considered and stochastic simulations over an extended period were performed to provide a measure of risk. A more sophisticated model could be built including all possible products and services, but this would be a time consuming exercise and it may be better to break the possible processes into smaller components and then attempt to optimize within each. Once optimal choices have been made within the various components some assessment of the combined strategy can be made.

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