

MISG 2011, Problem 1: Coal Mine pillar extraction

Group 1 and 2

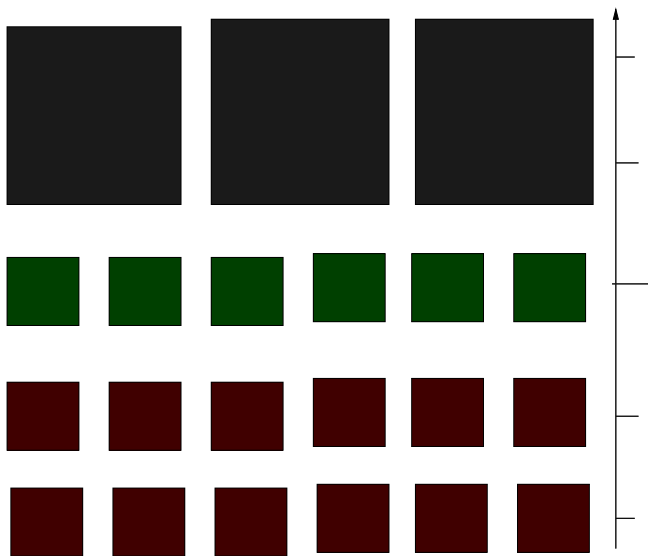
January 14, 2011

Group members

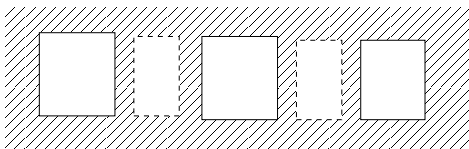
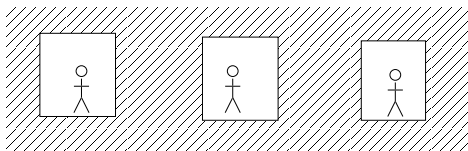
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A small coal mining development



Introduction



- Cutting pillars to make snooks
- QUESTION: How do snooks collapse in order to have a safe and controlled collapse of the rock?

Two approaches

- How do snooks collapse? “ The pillar problem”
- How stable is the roof when snooks collapse 'strut-beam analysis'

The set up

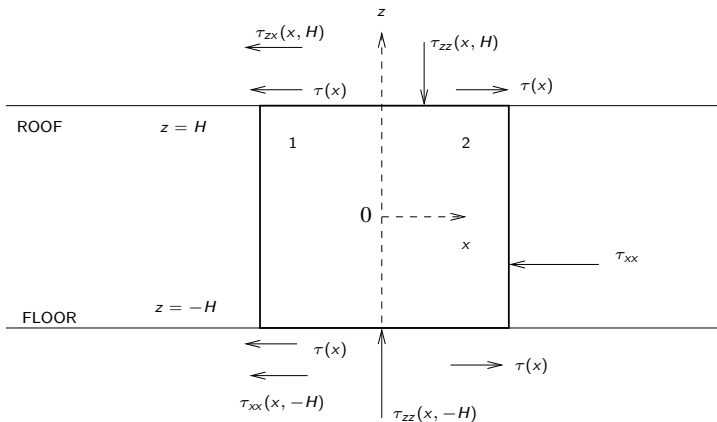


Figure: The stresses in a pillar

Governing equations

The stresses are considered in the xz -plane in the form

$$\tau_{xx} = \tau_{xx}(x, z), \quad \tau_{xz} = \tau_{xz}(x, z), \quad \tau_{zz} = \tau_{zz}(x, z)$$

and the average stress is defined as

$$\bar{\tau}_{ik}(x) = \frac{1}{2H} \int_{-H}^H \tau_{ik}(x, z) dz.$$

The equations of static equilibrium in 2D reads

$$\frac{\partial}{\partial x} \tau_{xx} + \frac{\partial}{\partial z} \tau_{zx} = 0 \quad (1)$$

$$\frac{\partial}{\partial x} \tau_{xz} + \frac{\partial}{\partial z} \tau_{zz} = 0 \quad (2)$$

Taking the averages in the previous equations, we arrive at

$$\frac{d}{dx} \bar{\tau}_{xx} + \frac{1}{2H} [\tau_{zx}(x, H) - \tau_{zx}(x, -H)] = 0 \quad (3)$$

$$\frac{d}{dx} \bar{\tau}_{xz} + \frac{1}{2H} [\tau_{zz}(x, H) - \tau_{zz}(x, -H)] = 0 \quad (4)$$

Governing equations and BCs for Region 1

- For $-1 \leq \bar{x} \leq 0$,

$$\frac{d^2}{d\bar{x}^2} \bar{\tau}_{xx}^{(1)} + \frac{3L}{\mu H} \frac{d}{d\bar{x}} \bar{\tau}_{xx}^{(1)} + 3m \left(\frac{L}{H} \right)^2 \bar{\tau}_{xx}^{(1)} = -3 \left(\frac{L}{H} \right)^2 \quad (5)$$

with

$$m = \left[(1 + \mu^2)^{\frac{1}{2}} + \mu \right]^2 > 1 \quad (6)$$

and μ the coefficient of internal friction, $\bar{x} = \frac{x}{L}$.

$$\bar{\tau}_{zz} = c_0 + \mu \bar{\tau}_{xx}.$$

and the boundary condition

$$\bar{\tau}_{xx}^{(1)}(-1) = 0 \quad \frac{d\bar{\tau}_{xx}^{(1)}}{d\bar{x}}(0) = 0. \quad (7)$$

Governing equations and BCs for Region 2

- For $0 \leq \bar{x} \leq 1$,

$$\frac{d^2}{d\bar{x}^2} \bar{\tau}_{xx}^{(2)} - \frac{3L}{\mu H} \frac{d}{d\bar{x}} \bar{\tau}_{xx}^{(2)} + 3m \left(\frac{L}{H} \right)^2 \bar{\tau}_{xx}^{(2)} = -3 \left(\frac{L}{H} \right)^2 \quad (8)$$

with the boundary condition

$$\bar{\tau}_{xx}^{(2)}(1) = 0. \quad (9)$$

Moreover, the following compatibility conditions hold

$$\bar{\tau}_{xx}^{(1)}(0) = \bar{\tau}_{xx}^{(2)}(0) \quad (10)$$

$$\frac{d\bar{\tau}_{xx}^{(2)}}{d\bar{x}}(0) = 0. \quad (11)$$

Solution of the Mohr-Coulomb constitutive equations

From the characteristic equations of the ODEs, we get the critical value of μ ,

$$\mu_{\text{crit}} \doteq \sqrt{\frac{3}{4(1 + \sqrt{3})}} \approx 0.52394565828751.$$

- $\mu < \mu_{\text{crit}}$,

$$\bar{\tau}_{xx}^{(1)} = \frac{1}{m} \frac{\lambda_2 \exp(-\lambda_1 \bar{x}) - \lambda_1 \exp(-\lambda_2 \bar{x})}{\lambda_2 \exp(\lambda_1) - \lambda_1 \exp(\lambda_2)} - \frac{1}{m}, \quad (12)$$

and

$$\bar{\tau}_{xx}^{(2)} = \frac{1}{m} \frac{\lambda_1 \exp(\lambda_2 \bar{x}) - \lambda_2 \exp(\lambda_1 \bar{x})}{\lambda_1 \exp(\lambda_2) - \lambda_2 \exp(\lambda_1)} - \frac{1}{m}, \quad (13)$$

Solution of the Mohr-Coulomb constitutive equations

- If $\mu = \mu_{\text{crit}}$,

$$\bar{\tau}_{xx}^{(1)} = \frac{1 + \alpha \bar{x}}{m(1 - \alpha)} e^{- (1 + \bar{x})} - \frac{1}{m}, \quad (14)$$

and

$$\bar{\tau}_{xx}^{(2)} = \frac{1 - \alpha \bar{x}}{m(1 - \alpha)} e^{- (1 - \bar{x})} - \frac{1}{m}, \quad (15)$$

- If $\mu > \mu_{\text{crit}}$,

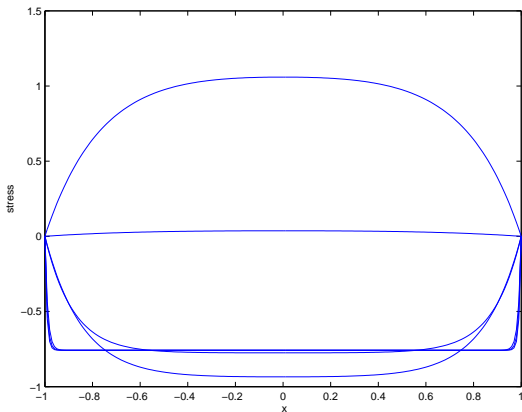
$$\bar{\tau}_{xx}^{(1)} = \frac{\tilde{\beta} \cos \tilde{\beta} \bar{x} + \alpha \sin \tilde{\beta} \bar{x}}{m(\tilde{\beta} \cos \tilde{\beta} - \alpha \sin \tilde{\beta})} e^{- (1 + \bar{x})} - \frac{1}{m}, \quad (16)$$

and

$$\bar{\tau}_{xx}^{(2)} = \frac{\tilde{\beta} \cos \tilde{\beta} \bar{x} - \alpha \sin \tilde{\beta} \bar{x}}{m(\tilde{\beta} \cos \tilde{\beta} - \alpha \sin \tilde{\beta})} e^{- (1 - \bar{x})} - \frac{1}{m}. \quad (17)$$

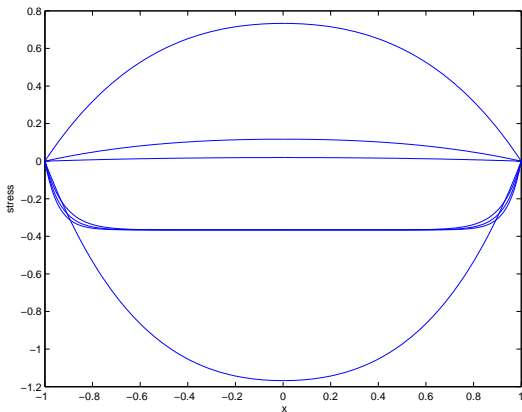
Analytical solutions

Case 1



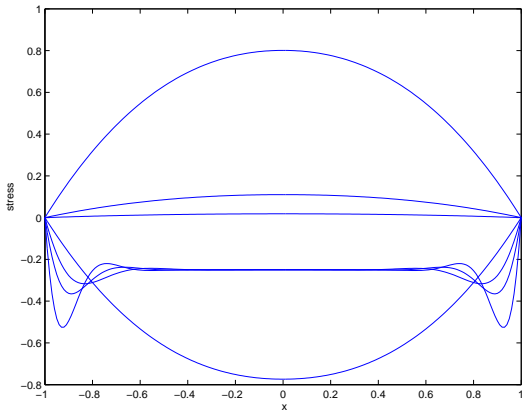
Analytical solutions

Case 2

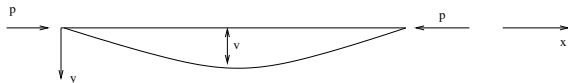


Analytical solutions

Case 3



Buckling of a Strut



- Two equations for the *bending moment* at point x along the *strut*

$$EI \frac{d^2 v}{dx^2} = M(x), \quad M(x) = -pv(x). \quad (18)$$

- These equations gives

$$\frac{d^2 v}{dx^2} + \frac{p}{EI} v(x) = 0. \quad (19)$$

- Solving (19) with the corresponding boundary conditions

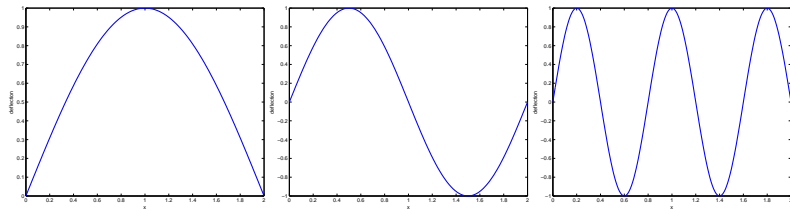
$$v(0) = 0, \quad v(L) = 0,$$

gives

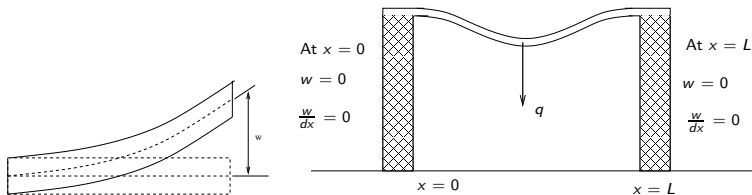
$$v(x) = A \sin\left(\frac{n\pi}{L}x\right), \quad n = 1, 2, 3, \dots \quad (20)$$

Buckling of a Strut

Numerical results



Euler-Bernoulli Beam



$$\frac{d^2}{dx^2} \left(EI \frac{d^2 w}{dx^2} \right) = q \quad (21)$$

Assumptions

- Mass is uniformly distributed.
- The beam is composed of an isotropic material.

$$-EI \frac{d^2 w}{dx^2} = M \quad \text{Bending moment} \quad (22)$$

$$-\frac{d}{dx} \left(EI \frac{d^2 w}{dx^2} \right) = Q \quad \text{Shear force on the beam} \quad (23)$$

Euler-Bernoulli Beam

- Now the deflection of the beam satisfies the equation

$$\frac{d^4 w}{dx^4} = \frac{q}{EI} \quad (24)$$

- subject to the boundary conditions

$$w(0) = w(L) = 0 \quad \frac{dw}{dx}(0) = \frac{dw}{dx}(L) = 0 \quad (25)$$

- The solution is found as

$$w(x) = \frac{q}{24EI} x^2 (x - L)^2. \quad (26)$$

- The curvature is given by

$$C = \left| - \frac{q L^2 - 6 q x L + 6 q x^2}{12} \right|$$

- A plot of C reveals that the beam always breaks at the end points.

A combined beam and strut

Motivation

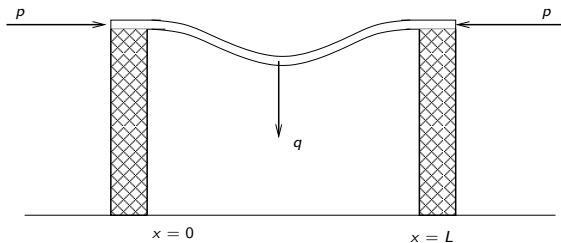


Figure: A combined beam and strut

A combined beam and strut

Governing equations

- Use the Euler-Bernoulli equation and the theory of Euler strut.
- For a unified model, we use the previous two models to have the equation

$$\frac{d^4 w}{dx^4} + \frac{p}{EI} \frac{d^2 w}{dx^2} = -\frac{q}{EI}. \quad (27)$$

- with the boundary conditions

$$\begin{aligned} w(0) = 0, \quad \frac{dw}{dx}(0) = 0 \text{ at } x = 0; \\ w(L) = 0, \quad \frac{dw}{dx}(L) = 0 \text{ at } x = L. \end{aligned} \quad (28)$$

- The nondimensional form of the model is

$$\frac{d^4 \bar{w}}{d\bar{x}^4} + \frac{pL^2}{EI} \frac{d^2 \bar{w}}{d\bar{x}^2} = -\frac{qL^4}{EIs}. \quad (29)$$

A combined beam and strut

Governing equations

- Define the so called *Mason number* M and s such that

$$M^2 = \frac{pL^2}{EI}, \quad s = \frac{qL^4}{EI} \quad (30)$$

- The problem reduces to

$$\frac{d^4 \bar{w}}{d\bar{x}^4} + M^2 \frac{d^2 \bar{w}}{d\bar{x}^2} = -1. \quad (31)$$

- with the boundary conditions

$$\begin{aligned} \bar{w}(0) = 0, \quad \frac{d\bar{w}}{d\bar{x}}(0) = 0 \text{ at } x = 0; \\ \bar{w}(1) = 0, \quad \frac{d\bar{w}}{d\bar{x}}(1) = 0 \text{ at } x = 1. \end{aligned} \quad (32)$$

A combined beam and strut

Solution to the Governing equations

- The solution of the previous equation is found as

$$\bar{w}(x) = \frac{1}{2M^2} \left(\frac{1}{4} - \left(x - \frac{1}{2}\right)^2 \right) + \frac{1}{2M^3 \sin \frac{M}{2}} \left(\cos \frac{M}{2} - \cos \left(M \left(x - \frac{1}{2} \right) \right) \right) \quad (33)$$

- The curvature is measured with the help of

$$|\bar{w}''(x)| = \left| -\frac{1}{M^2} + \frac{\cos \left(M \left(x - \frac{1}{2} \right) \right)}{2M \sin \frac{M}{2}} \right|.$$

A combined beam and strut

Numerical Solution to the Governing equations

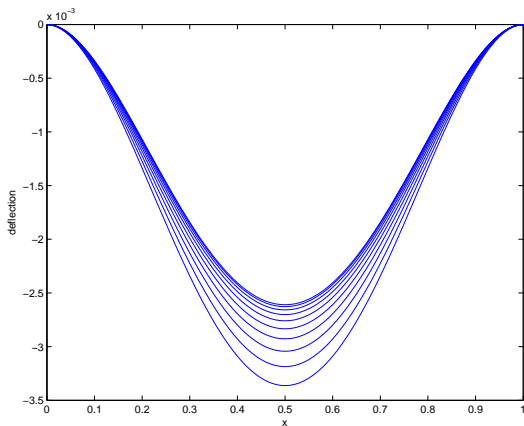


Figure: A plot of the deflection for different values of M , $M = 0.3 - 3$.

A combined beam and strut

Numerical Results

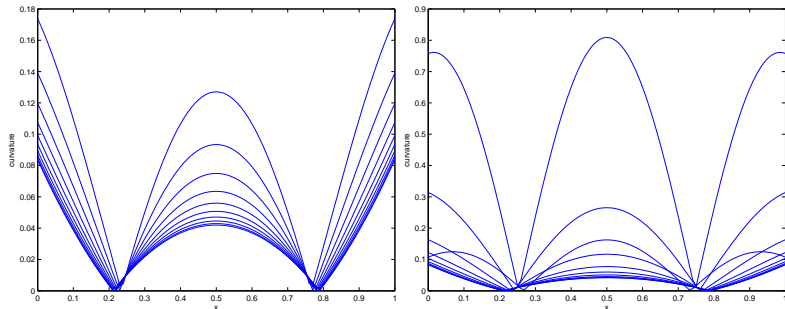


Figure: A plot of the curvature $|w''(x)|$ for different values of M .

A combined beam and strut

Numerical Results

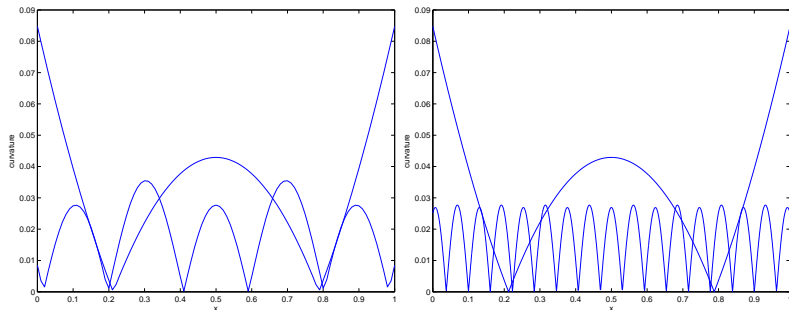


Figure: A plot of the curvature $|w''(x)|$ for different values of M .

A combined beam and strut

Numerical Results

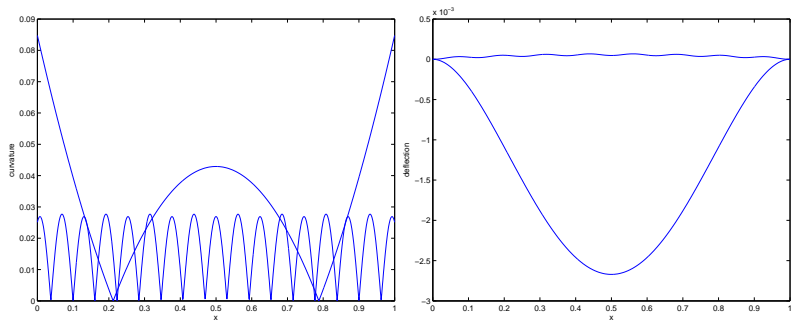


Figure: A plot of the deflection and the curvature $M = 0.8$ and $M = 8$.

A combined beam and strut

Important results

- It appears that for small ($M < 2\pi$) values of M , the beam breaks at the end points.
- We never recover the first mode of the solution of the Euler strut equation.
- This is due to the fact that for the beam we have 4 boundary conditions and only 2 for the Euler strut. The equivalent fourth order ODE associated with the Euler strut has different boundary conditions.

Conclusions

- Snook collapse
 - Stress is dependent of the quantity $\frac{L}{H}$. This ration of height to width of the pillar is crucial. In general mining practices, $\frac{L}{H} \geq 5$; but for secondary mining operations, we can have $\frac{L}{H} < 5$.
 - In this later case, the stress is confined to boundary layers and the problem correspond to singularly perturbed problem.
- Roof stability
 - We found a critical ratio, M , for the roof to buckle or to break.
 - When $M < 2\pi$, what happen in most of the cases, the roof always break next to the snook.

Thank you

Thank you

THANK YOU