Management of Harvesting of Rhinos

Study Group Report

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Outline

- Currently there are about 2000 black rhinos in South Africa.
- The goal is to maximise the growth rate of the rhino population.

Aims

Primary Objectives

- Develop a 2 population harvesting model.
- Try to determine the optimal amount of rhinos to be harvested.

Assumptions

We make the following assumptions

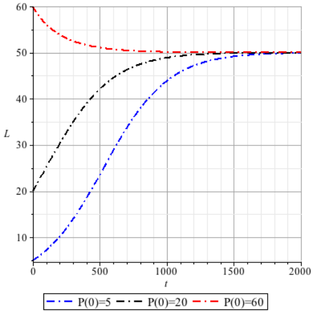
- Disregard gender, infectious diseases, poaching etc.
- A constant carrying capacity.

A Single Population Framework

Logistic Equation

$$\frac{dP}{dt} = RP(t)\left(1 - \frac{P(t)}{K}\right), \qquad P(0) = P_0 \tag{1}$$

Figure : Logistic Model



A Single Population Framework

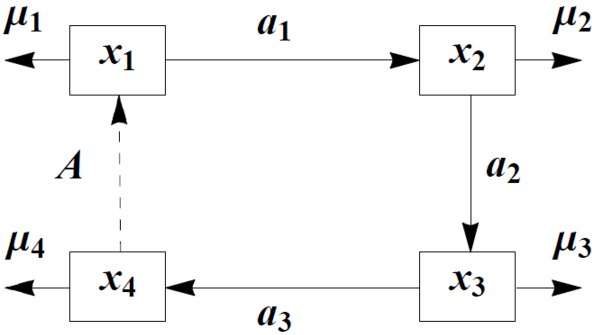
Allee Effect

$$\frac{dP}{dt} = RP(t)\left(1 - \frac{P(t)}{K}\right)\left(\frac{P(t)}{\mu} - 1\right), \qquad P(0) = P_0. \tag{2}$$

A Single Population Framework

Age Structure

Figure : Single Population Model



A Single Population Framework

Age Structure

$$\frac{dx_1}{dt} = Ax_4 - x_1(a_1 + \mu_1), \tag{3}$$

$$\frac{dx_2}{dt} = a_1 x_1 - x_2(a_2 + \mu_2), \tag{4}$$

$$\frac{dx_3}{dt} = a_2 x_2 - x_3(a_3 + \mu_3 + h_3), \tag{5}$$

$$\frac{dx_4}{dt} = a_3 x_3 - (\mu_4 + h_4)x_4. \tag{6}$$

Interaction Between 2 Population with Harvesting

Mathematical Model

Assumptions

- Consider only two populations
- No naturally occuring direct interaction
- Constant carrying capacity and recruitment rate
- Harvesting only to relocate rhino to other population

Interaction Between 2 Population with Harvesting

Mathematical Model

Variables and Parameters

- P_1 and P_2 are the rhino populations in area 1 and 2, respectively;
- μ_1 and μ_2 are recruitment rate for rhino populations in area 1 and 2, respectively;
- K is the carrying capacity in P_1, and β is the carryng capacity for P_2 dependent on P_1
- h is the harvesting rate and α is the portion of harvested rhino succesfully entering population P_2.

Interaction Between 2 Population with Harvesting

Mathematical Model

Dynamical Equations

$$\frac{dP_1}{dt} = \mu_1 P_1 \left(1 - \frac{P_1}{K}\right) - hP_1 \quad (7)$$

$$\frac{dP_2}{dt} = \mu_2 P_2 \left(1 - \frac{P_2}{\beta K}\right) + \alpha h P_1. \quad (8)$$

Interaction Between 2 Population with Harvesting

Nondimensinal equations

Nondimensional equations

$$\frac{dx}{d\tau} = x(1-x) - \sigma x \quad (9)$$

$$\frac{dy}{d\tau} = \mu_o y\left(1-\frac{y}{\beta}\right) + \alpha\sigma x \quad (10)$$

with

$$x = \frac{P_1}{K}$$

$$y = \frac{P_2}{K}$$

$$\tau = \mu_1 t$$

and

$$\mu_o = \frac{\mu_2}{\mu_1}$$

$$\sigma = \frac{K}{\mu_1}$$

Interaction Between 2 Population with Harvesting

Equilibrium points

Equilibrium points

$$E_0 = (0, 0),$$

Interaction Between 2 Population with Harvesting

Equilibrium points

Equilibrium points

$$E_0 = (0,0), \quad E_1 = (0, K_2)$$

Interaction Between 2 Population with Harvesting

Equilibrium points

Equilibrium points

$$
\begin{aligned}
E_0 &= (0,0), \quad E_1 = (0, K_2) \\
E_2 &= \left(\frac{K_1 (\mu_1 - h)}{\mu_1}, \right. \\
&\quad \left. 1/2 \frac{\left(\mu_2 K_2 + \sqrt{\frac{\mu_2 K_2 (\mu_2 K_2 \mu_1 + 4\, h\alpha\, K_1 \mu_1 - 4\, h^2 \alpha\, K_1)}{\mu_1}} \right)}{\mu_2} \right)
\end{aligned}
$$

Interaction Between 2 Population with Harvesting

Equilibrium points

Equilibrium points

$$E_0 = (0,0), \quad E_1 = (0, K_2)$$

$$E_2 = \left(\frac{K_1(\mu_1 - h)}{\mu_1}, \; 1/2 \frac{\left(\mu_2 K_2 + \sqrt{\frac{\mu_2 K_2 (\mu_2 K_2 \mu_1 + 4\,h\alpha\,K_1 \mu_1 - 4\,h^2 \alpha\,K_1)}{\mu_1}}\right)}{\mu_2} \right)$$

with

$$G = \frac{\mu_2 K_2 \mu_1 + 4\,h\alpha\,K_1 \mu_1}{4 h^2 \alpha\,K_1} > 1$$

Interaction Between 2 Population with Harvesting

Local stability

Local stability

$$J = \begin{bmatrix} \mu_1\left(1-\frac{x}{K_1}\right)-\frac{\mu_1 x}{K_1}-h & 0 \\ h\alpha & \mu_2\left(1-\frac{y}{K_2}\right)-\frac{\mu_2 y}{K_2} \end{bmatrix} \quad (11)$$

Local stability

- E_0 with eigenvalues $(\mu_1 - h, \mu_2)$
- E_1 with eigenvalues $(\mu_1 - h, -\mu_2)$
- E_2 with eigenvalues
  $$\left(\mu_1 - h, -\frac{\sqrt{{\mu_2}^2{K_2}^2+4\,K_1\alpha\, hK_2\mu_2-4\,\frac{\mu_2 K_2 h^2\alpha\, K_1}{\mu_1}}}{K_2}\right),$$

Interaction Between 2 Population with Harvesting

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Local stability

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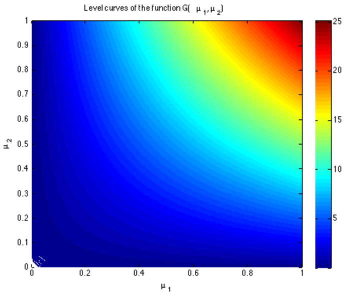
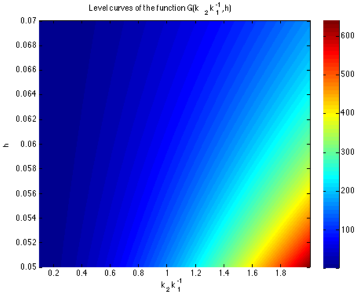
- E_2 with eigenvalues

$$\left(\mu_1 - h, -\frac{\sqrt{{\mu_2}^2{K_2}^2+4\,K_1\alpha\,hK_2\mu_2-4\,\frac{\mu_2 K_2 h^2\alpha\,K_1}{\mu_1}}}{K_2}\right), G > 1.$$

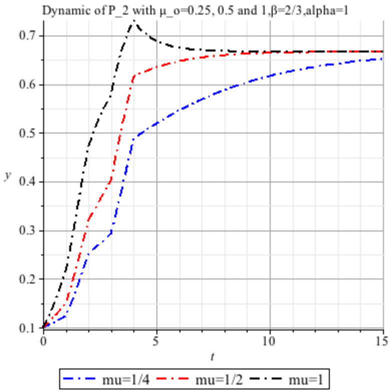
Numerical simulation

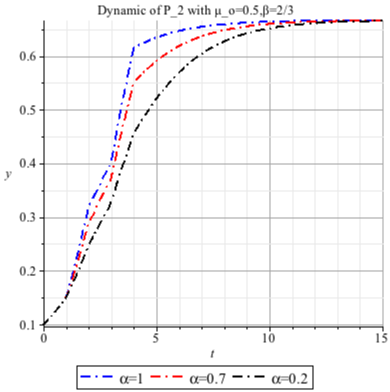
$$G = \frac{\mu_2 K_2 \mu_1 + 4\,h\alpha\,K_1 \mu_1}{4h^2 \alpha\,K_1}.$$



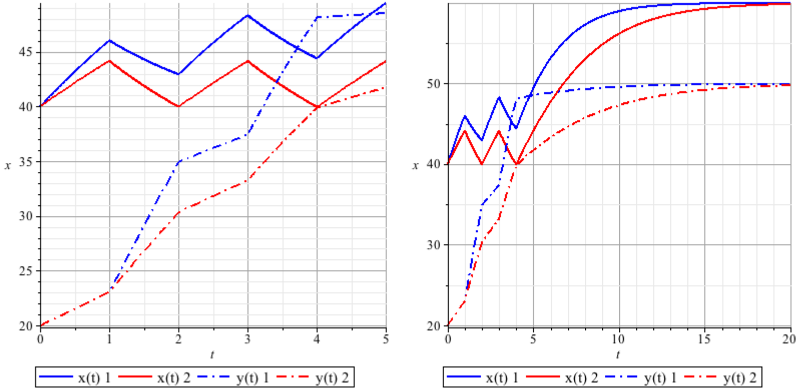


Numerical simulation





Numerical simulation



Further Work

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Is the effort to relocate 10 rhinos to P_2 with initial population $P_2(0) = 50$, $K_2 = 70$ the same as the effort to relocate them to P_3 with initial condition $P_3(0) = 20$, $K_3 = 70$?

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Proposed Equations

$$
\begin{aligned}
\frac{dP_1}{dt} &= \mu_1 P_1 \left(1 - \frac{P_1}{K_1}\right) - hP_1 \\
\frac{dQ}{dt} &= hP_1 - \sigma Q P_2 \\
\frac{dP_2}{dt} &= \mu_2 P_2 \left(1 - \frac{P_2}{K_2}\right) + r\sigma Q P_2 - \gamma P_2 Q \\
\frac{dR}{dt} &= (1 - r)\sigma Q P_2 + \gamma P_2 Q.
\end{aligned}
$$

The End