# A METHOD FOR STOCHASTIC ESTIMATION OF COST AND COMPLETION TIME OF A MINING PROJECT

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**Abstract**

The standard methodology used for estimating the cost and completion time of a mining project is inadequate and sometimes produces misleading time and cost estimates. This is due to the fact that the methodology used does not capture the stochastic nature of the project activities sufficiently accurately. In this study we design a Monte-Carlo based procedure that provides reliable time and cost estimates for the project if the activity durations follow known probability distributions. The significance of this problem has been elevated recently as financiers of mining projects have started to resort to litigation against project proponents, claiming that they were mislead into investing in projects that would be completed neither on time nor within budget.

## 1 Introduction

Before a mine can be brought into full production it must first go through a project phase. The mining project phase can broadly be broken up, in order of precedence, level of detail and degree of confidence in estimates, into Scoping Study, Pre-Feasibility Study and finally Bankable Feasibility Study (BFS). The BFS report is the document used to motivate funding from financing institutions and the company's Board of Directors, or for raising funds from the stock markets. Central to the process is the analysis and estimation of project cost and completion time, and

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the risk associated with making these estimates. In most projects there is a cost and time over-run, in which case the project managers resort to recalculating the project cost and completion time. Consequently project managers have to go back to the Board or stock markets to motivate for more funding to complete the project, based on the recalculated estimates. However, in recent times financiers of such projects have started resorting to litigation against the project proponents, claiming that they were misled into investing in a project that would not be completed on time and within budget. A case in point is the shareholder class action lawsuit filed against NovaGold over the Galore Creek copper copper–gold project in which costs had been revised to 127% greater than the initial estimates and the project was two and a half years behind schedule [1–3].

Project mining teams claim that the source of the completion time and cost errors lies in the methods used for estimating the cost and completion time of the project [1–3]. The standard methodology that is used in evaluating projects is PERT/CPM[1]. This is the methodology that is embedded in project management software such as Microsoft Project or Enhanced Production Scheduler [4]. PERT/CPM assumes that a project is made up of inter–linked activities that occur as a series–in–parallel network, from start to finish [5]. We will use the activity-on-node representation of a project in which each node represents an activity and each directed arc represents a precedence relation between two activities, as shown in Figure 1 [4]. Figure 1 represents a project made up of seven activities or nodes; nodes 0 and 8 are dummy start and end activities respectively. Both of the dummy nodes are assumed to have a deterministic duration of zero. As noted above, the edges in the graph show the precedence relations between activities. For example, the edge between activities 1 and 3 tells us that activity 3 cannot start until the activity 1 is completed.

The main problem with the CPM is that it is deterministic and does not capture the stochastic nature of the cost and time estimates of project activities [4]. Now PERT incorporates the stochastic nature of the cost and completion time estimates of each activity based on the estimates of the most likely duration, the duration assuming the most favourable conditions and the duration assuming the least favourable conditions of each activity [4]. However, it has a number of limitations which will be discussed in more detail in subsection 2.2. There is a need for a general method that provides good time and cost estimates for a project in which the duration of each activity follows a known probability distribution. This paper presents a Monte-Carlo based method to address this stochastic situation.

The rest of the paper is organised as follows. In section 2, we present the CPM and PERT methodologies together with their limitations. Our proposed Monte-

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[1] PERT stands for Project Evaluation and Review Technique. CPM stands for Critical Path Method.

Carlo method for mining projects together with some computational results are presented in section 3. Concluding remarks are made in section 4.

# 2 CPM and PERT

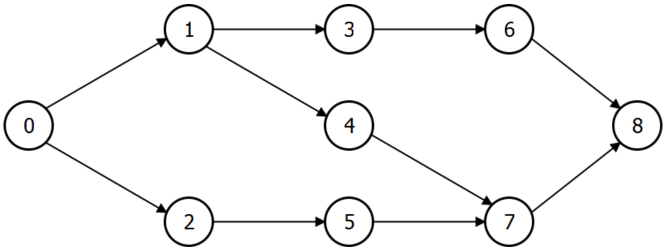

Figure 1: Project network of the mining project

In this section, we give a brief overview of the CPM and PERT methodologies. Prior to this, we give some important definitions. These definitions are taken from [5].

**Definition 2.1** (Network)**.**
*A project network is a graphic that shows all the activities in the project and the order in which they have to be executed.*

The project network for the mining project considered in this paper is shown in Figure 1.

**Definition 2.2** (Critical path)**.**
*The critical path is the longest path in the project network.*

## 2.1 CPM

The details of the CPM given in this section are taken from [4] and [5]. One of the primary goals of the CPM is to determine the critical path through the network. When applying the CPM, we first determine the earliest time that each activity in the network can start and finish. This is done by making what is called a *forward pass* (working from the left to the right of the network) through the network. The forward pass also determines the earliest time that the project itself can be completed. Secondly, we make a *backward pass* (working from the right to the left of the network) through the network to determine the latest time that each activity can start and end without delaying the completion of the project found using the forward pass. Activities are said to be on the critical path if their earliest and latest start times are equal. Clearly any delay in the start or finish times of the activities on the critical path delays the completion of the project. According to the CPM one should always focus on and avoid delays in the activities on the critical path.

On the forward pass the earliest start time of each activity $i$ is given by

$$EST_i = \max_j(EFT_j),$$

where $j$ is any activity that is linked to activity $i$ by an edge from $j$ to $i$ and $EFT_j$ is the earliest finish time of activity $j$. The earliest finish time of activity $i$ is given by

$$EFT_i = EST_i + t_i.$$

The earliest start and finish times of the starting activity are set to zero.

On the backward pass the latest start time of activity $i$ is given by

$$LST_i = LFT_i - t_i,$$

where $LFT_i$ is the latest finish time of activity $i$ which is given by

$$LFT_i = \min_j(LST_j).$$

Note that the latest finish time of the final activity is set to be the same as its earliest finish time since it has a duration of zero.

The slack of activity $i$, $s_i$, is given by

$$s_i = LST_i - EST_i = LFT_i - EFT_i.$$

The critical activities are the activities with zero slack. The estimated completion time of the project is the sum of the durations of the critical activities.

The main problem with the CPM is that it assumes that the durations of the activities are deterministic, this is hardly ever the case in reality. Further details about the implementation of the CPM and problems with the method can be found in [4].

## 2.2 PERT

The details of PERT given in this section are taken from [4] and [5]. The underlying idea of PERT is similar to that of CPM. The only difference is that PERT assumes that the activity durations are random variables that follow a beta distribution. PERT requires three estimates for each activity in the project. Specifically, for each activity $i$, PERT requires; an estimate of the duration of activity $i$ assuming the most favourable conditions, an estimate of the duration of activity $i$ assuming the least favourable conditions and an estimate of the most likely duration of activity $i$. We denote these quantities by $a_i$, $b_i$ and $m_i$ respectively. These quantities can be thought of as representing the best case, the worst case and the most likely time required to perform activity $i$. PERT uses these estimates to calculate the expected duration $t_i$ of each activity together with the variance $v_i$ associated with this duration as follows

$$\begin{aligned} t_i =& \frac{a_i + 4m_i + b_i}{6}, \\ v_i =& \frac{(b_i - a_i)^2}{36}. \end{aligned}$$

These formulae are based on the assumption that the duration of each activity follows a beta distribution. Using these estimates, PERT identifies the critical path and critical activities using a similar procedure to the CPM.

PERT has a number of limitations. Firstly it assumes that the activity durations follow beta distributions which will not always be the case. PERT also assumes that the activity durations are independent random variables. This is generally not a realistic assumption; for example if one activity along the critical path runs over its expected time, the project manager ought to make sure that one or more subsequent activities run within their respective expected time to ensure that the project does not run over time. As a result of this, the variance of the activities following the delayed activity in the network will be reduced. Another major problem with PERT is that the critical path is defined as the path with the longest expected completion time. The project manager will then focus on the activities on the critical path, believing that they are the activities most likely to delay the completion of the project. In reality, activities which are not on the critical path but which have a high variance can pose a greater risk of delaying the project. Further information relating to PERT can be found in [4].

## 3 Monte-Carlo Simulation

Although PERT has major drawbacks, it does highlight an important point, namely the stochastic nature of the activity durations. However, as noted in subsection 2.2, activities not on PERT's critical path might be more important to the completion of the project than the activities that PERT identifies as critical. Indeed, many activities in the project could have a probability of becoming critical during project execution. This probability could be close to 0 or 1. The mining project manager should focus on the activities with the highest probability of being critical, regardless of whether they fall on the critical path.

| Activity | Distribution | Parameters |
|---|---|---|
| 1 | Normal | $(\mu, \sigma) = (2, 0.2)$ |
| 2 | Beta | $(x; \alpha, \beta) = (24; 1, 4)$ |
| 3 | Beta | $(x; \alpha, \beta) = (12; 4, 1)$ |
| 4 | Uniform | $(\min, \max) = (0, 2.5)$ |
| 5 | Log-normal | $(\mu, \sigma) = (2, 0.2)$ |
| 6 | Normal | $(\mu, \sigma) = (5, 2.5)$ |
| 7 | Beta | $(x; \alpha, \beta) = (7.2; 2, 3)$ |

Table 1: The probability distributions used for the durations of the activities.

The best way of evaluating the impact of variability in activity duration on the completion time of the project involves the Monte-Carlo simulation technique. The basic idea of this technique is similar to that of PERT, except that here we do not focus on the critical path, but rather on the probability of each activity being critical. In the Monte-Carlo simulation technique we assume that each activity duration is a random variable following some probability distribution. To apply this technique we first determine an approximate duration for each activity by sampling from its duration distribution. Using these estimated durations we determine the critical path of the project using the method described in subsection 2.1. We now repeat this process many times (for our numerical experiments we used $10^5$ simulations) and determine the frequency with which each activity falls on the critical path. These frequencies give us the probability that an activity will lie on the critical path.

The Monte-Carlo simulation technique was applied to the project in Figure 1. The duration distributions used for each activity are given in Table 1. We see that the duration distributions are no longer restricted to beta distributions. The results obtained are given in Table 2. Table 2a gives the probability of each activity being critical. Table 2b gives information about the completion time of the project. It can be seen in Table 2b that the project as described above can be completed within 41.7 (or 42) time periods with a certainty of 99.7%. From the results in Table 2a, the project manager should focus on activities 1, 2, 3, 5, 6 and 7, since these are the activities that are likely to be critical.

| Activities | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Probabilities (%) | 93.7 | 100 | 93.6 | 6.3 | 100 | 100 | 100 |

(a) Probability of each activity being on the critical path

| | |
|---|---|
| Average | 36.1 |
| Variance | 5.04 |
| Minimum | 8.3 |
| Maximum | 49.4 |
| 99.7% of prob. of completion by | 41.7 |

(b) Completion time

Table 2: The computational results obtained using the Monte-Carlo simulation technique.

# 4 Conclusion

We have presented a procedure which provides extensive information about the completion time of a mining project whose activities have know duration distributions. The procedure makes use of Monte-Carlo simulation to determine the probability that an activity will lie on the critical path. This allows the project manager to concentrate on activities which are likely to become critical during project execution. In addition the Monte-Carlo simulation allows us to calculate the probability that the project will be completed within a certain time. This method overcomes the problems encountered when using PERT/CPM to estimate the project duration and should allow mining project managers to supply investors with more accurate estimates of project cost and completion time.

# References

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