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PREFACE

The Fifth Mathematics in Industry Study Group (MISG) Workshop in South Africa was held in the School of Computational and Applied Mathematics at the University of the Witwatersrand, Johannesburg, from Monday 28 January to Friday 1 February 2008.

There were fifty-three participants at the MISG. Ten University staff, thirty-three postgraduate students, five Industry Representatives and five invited overseas guests attended. The guests were:

Dr Neville Fowkes:	University of Western Australia, Australia
Dr Dhananjay Vakaskar:	University of Baroda, Vadorara, India
Prof João Freitas:	Technical University of Lisbon, Portugal
Dr Jean Charpin	University of Limerick, Ireland
Prof Steven Damelin	Georgia Southern University, USA

The South African Universities which were represented were:

University of Cape Town
University of Johannesburg
University of Kwa Zulu-Natal
University of Limpopo
North West University
University of Pretoria
University of Stellenbosch
Tshwane University of Technology
Vaal University of Technology
University of the Witwatersrand

One University from the rest of Africa was represented:
University of Zimbabwe

The MISG Workshop was opened by Professor Ramesh Bharuthram, the Dean of the Faculty of Science at the University of the Witwatersrand.

The MISG Workshop followed the established format for MISG meetings held in the United Kingdom, Australia, New Zealand, Canada, Asia and the United States. South African industry had been approached to submit problems during the second half of 2007. Six problems were submitted. Five of the problems were investigated at the MISG. The choice was determined by the interests and experience of the participants. On Monday morning each Industry Representative made a twenty-five minute presentation in which he described the problem and outlined what he thought needed to be done. On Tuesday, Wednesday and Thursday the academics together with the graduate students, worked in small study groups on problems which suited their interest and expertise. Each problem was coordinated by a senior moderator and a student moderator. The role of the senior moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderator was to present the fifteen minute progress report on Wednesday afternoon. The moderators were in contact with the Industry Representatives on Tuesday, Wednesday and Thursday. On Friday morning there was a full report back session to industry. Each senior moderator made a twenty-five minute presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes in which to make comments on the progress and results which were reported. The MISG ended at lunch time on Friday.

Three invited lectures were given, on Tuesday, Wednesday and Thursday morning, by applied mathematicians with experience at solving problems from industry. The aim of the lectures was to show how mathematics could be used to solve problems in industry:

Prof Steven Damelin	*"Compression of hyperspectral image data"*
Prof João Freitas:	*"Hybrid finite element formulations"*
Prof Steven Damelin	*"Interdisciplinary mathematics at Georgia Southern University"*

The main contribution made during the week of the MISG was to expose the industrial problems to the mathematics community and to do modelling and simulations. Work continued on the problems after the meeting ended. In March 2008 an equation-free Executive Summary, not more than two pages in length, for each problem was given to each Industry Representative. The Executive Summary was designed to inform Management of the progress made at the MISG on their problem. In the Proceedings of the MISG the mathematical progress made on each problem up to December 2008 is presented and suggestions for further work are made. Moderators with the most active members of their group and the Industry Representative will be encouraged to publish their results in international journals.

A MISG brings together mathematicians to work on and solve research problems of industrial origin. Mathematical solutions will assist South African industry to become more efficient and competitive thereby creating jobs and contributing to the prosperity of South Africa. Mathematicians in turn see the challenges facing industry. By working in small groups with experienced industrial mathematicians academics receive training in solving problems from industry. New collaborations are established within South Africa and also internationally with the invited guests. Higher degree students are encouraged to participate in the small study groups and the work done could develop into suitable mathematics in industry topics for Masters dissertations and PhD theses. By demonstrating to companies that mathematics can be used successfully to solve problems in industry, job opportunities will be created in industry for graduates in the mathematical sciences. Applied industrial problems can also lead to problems in basic research. Some of the problems should provide innovative teaching material since mathematical modelling plays a central role in the solution process.

The MISG was preceeded on Saturday 26 January 2008 by a Mathematics in Industry Graduate Student Workshop. Thirty three graduate students participated in the Workshop. The aim of the Workshop was to prepare the graduate students for the MISG which started the following Monday. The Facilitator of the Workshop was Dr Neiville Fowkes of the University of Western Australia. He was assisted by Professor Montaz Ali of the University of the Witwatersrand. The Workshop was officially opened by Professor Yunus Ballim, Deputy Vice-Chancellor (Academic) of the University of the Witwatersrand. Three problems were presented to the graduate students:

Temperature variations in a parked car
The undersea oil detection problem
Model of an active vehicle suspension system

The graduate students then spit into small study groups and worked on the problem of their choice throughout the day. One graduate student from each group presented the results at a report back session on Saturday evening.

The sponsors of the MISG were:

National Research Foundation (NFR), Pretoria, South Africa

Dean’s Discretionary Fund, Faculty of Science Research Committee, University of the Witwatersrand

School of Computational and Applied Mathematics, University of the Witwatersrand.

We thank the sponsors without whom the Mathematics in Industry Study Group meeting could not have taken place.

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Stacey, Richard – School of Mining Engineering, University of the Witwatersrand, Johannesburg.

PROBLEMS

For each problem submitted by industry, the title of the problem, the industry presenting the problem, the industry representatives and the academic moderators are listed below.

Problem 1.

Title: An investigation of data compression techniques for hyperspectral core imager data
Industry: Anglo American
Industry Representative: Michael Sears
Moderator: Steven Damelin
Student Moderators: Louis du Plessis and Michael Mitchley

Problem 2.

Title: Rockbursts and mud
Industry: Mining
Industry Representative: Richard Stacey
Moderators: Eunice Mureithi and Neville Fowkes

Problem 3.

Title: Time domain simulations of dynamic river networks
Industry: Optin, Optimal and Intelligent Solutions
Industry Representative: Tony Lange
Moderator: Mapundi Banda
Student Moderator: Jean-Medard Tchoukouegno Ngnotchouye

Problem 4.

Title: Wrinkling of paper labels on beer bottles
Industry: South African Breweries
Industry Representative: Thabo Ntlamelle
Moderator: Astri Sjöberg
Student Moderators: Terry-Leigh Oliphant and Warryn David

Problem 5.

Title: Modelling notch flow of viscous non-Newtonian fluids
Industry: Mining
Industry Representative: Paul Slatter
Moderator: Tim Myers

Executive Summaries

A brief description of each problem is given followed by the equation-free Executive Summary for the problem

AN INVESTIGATION OF DATA COMPRESSION TECHNIQUES FOR HYPERSPECTRAL CORE IMAGER DATA

Anglo American Corporation

Industry Representative

Michael Sears, School of Computer Science, University of the Witwatersrand, Johannesburg

Moderator

Steven Damelin, The Unit for Advances in Mathematics and its Applications, Department of Mathematical Sciences, GSU, P.O. Box 8093, Statesboro, GA 30460-8093, U.SA.

Description

Hyperspectral data sets tend to be very large. The Hyperspectral Core Imager (HCI) used by AngloGold Ashanti to produce hyperspectral images of core samples produces 2Gb of data in an hour. With thousands of metres of core samples available for processing it is easy to see that the amount of data available becomes unmanageable. The problem we were faced with was to provide a tractable method for the storage and/or analysis of this data.

Executive summary

The purpose of the study group was twofold. First, to investigate algorithms for tractable analysis of real hyperspectral data provided to us by the Geosciences Resource Group of Anglo Technical Division. Second, to involve graduate students in ongoing projects related to this work.

Three different approaches were considered. The first approach relied on extracting the most conspicuous features of every spectrum. This was done by finding the largest changes in gradients (around turning points). Enough information is stored in only these values to reconstruct a close approximation to the real spectrum.

The second approach exploited the redundancy in hyperspectral data with non-linear diffusion maps. This approach transforms the data to a lower-dimensional space, where it can be more easily analysed. Although it performed well, it makes excessive memory demands, since it involves the spectral decomposition of very large matrices.

Lastly, a wavelet approach was considered. An orthogonal Haar wavelet basis was used to map the spectral data for every pixel onto its approximating coefficients. This method is particularly fast and can be applied with only a single matrix multiplication.

To visualise the results of all three approaches a K-means clustering was performed on the results. All methods performed well and gave us a strong basis for future work. Further work will be done on the non-linear diffusion and wavelet approaches, and both will be used as honours projects by graduate students. In particular we wish to investigate the application of compressive sampling and parallelization to a diffusion map based approach and the effects of a more complicated basis to a wavelet based method.

ROCKBURSTS AND MUD

Mining Industry

Industry Representative

Richard Stacey, School of Mining Engineering, University of the Witwatersrand, Johannesburg

Moderators

Eunice Mureithi, Department of Mathematics and Applied Mathematics, University of Pretoria, Pretoria

Neville Fowkes, School of Mathematics and Statistics, University of Western Australia, Crawley, WA, Australia

Description

Rockburst are explosive events caused by the build up of stress in the walls of mining tunnels. Either directly because of this build-up or because of impulses of seismic or mining origin, rocks are ejected from the tunnel walls, or concrete is broken and heaved. It has been observed that such events do not seem to occur in the presence of water or mud on the tunnel floor. The objective was to investigate various mechanisms that could explain this.

Executive summary

The group discussed several possibilities but in the end the two most likely mechanisms were seen to be wave trapping (as suggested by Richard Stacey) and lubricative suction.

A positive pressure pulse propagating through rock is reflected at a free surface as a tensile pulse and, because rock is weak under tension, the surface may rupture causing a rock-burst. If however, the rock surface is lined with a rigid material then such a positive pressure pulse will be reflected as a positive pulse so that the rock surface will not be subjected to tension; the liner protects the surface from such an impact. A layer of water can be considered to act as a liner; the question is whether such a liner will provide sufficient protection to avoid a rockburst. It was found that a normally incident P wave experienced a phase change as a result of impact on a water liner the size of which depended on two dimensionless combinations of parameters, one related to the liner thickness and frequency of incoming wave and the other related to the relative propagation and mechanical characteristics of the liner and the rock. For sufficiently high frequency waves the water liner provides protection. Further work is being undertaken to assess the

situation if the incident wave is of general type (with P and S components) and is non-normally incident and also to see what happens if the liner breaks up. The results will be presented in the final report.

Naturally occurring rock is heavily fractured as a result of past crustal movements. The effect of an impulse can be to rupture remaining connections between a lump of rock material and the surrounding rock so that the lump is ejected from the wall. Water at the base of a tunnel may penetrate into the thin fractures (gravitationally and surface tension driven) and if this is the case then the thin 'lubricating' layer provides strong resistance (suction) to separation between the lump and the surrounding wall; a lubrication effect. A model was developed to assess this effect. It was found that the suction force can be very large for small separation distances and may inhibit rockburst. The investigation is continuing.

TIME DOMAIN SIMULATIONS OF DYNAMIC RIVER NETWORKS

Optin, Optimal and Intelligent Solutions

Industry Representative

Tony Lange, Optin, 9 Eldorado Road, Victory Park, Johannesburg

Moderator

Mapundi Banda, School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg

Description

The objective is to derive suitable models for rivers/dams and river-dam networks that take into account a time domain and the physical features of these assets: inflow, outflow, evaporation, infiltration/seepage, abstraction and possibly run-off or spill. In addition geometric attributes like length, cross-sectional area and surface area need to be included. The proposed model needs to be simulated on real life data and errors due to modelling and data collection rigorously approximated and quantified.

Executive summary

Depending on the accuracy and the efficiency needed the problem can be considered to have different scales. At the scale of a network, we need a macroscopic (coarse) model that describes the global dynamics of the river and dam networks; whereas at the scale of rivers and dams we need a microscopic (fine) model. In general such flows are modelled in a unifying framework of the open channel flows the most common of which is the St Venant Model. This model describes the hydrodynamics of the flow in which mass (wet cross-sectional area) and momentum (discharge) of the flow is conserved. The model also takes into account other forces such as friction forces, forces due to the bottom topography and forces due to non-uniform channel width of the water body. In this model inflow and outflow are defined as boundary conditions. Extensions to include evaporation, abstraction, seepage and runoff will be undertaken. Such a model would be appropriate at a fine scale. It can be used for benchmarking when a coarser model is needed. Numerical methods for such a model are abundant in the literature and an implementation of the so called central schemes gave us good qualitative results for simple flow examples.

For networks, a mass balance equation has been considered. In this approach an equation is derived in which the average flow velocity is approximated by

Manning's equation. To complete the mass balance model, phenomenon like evaporation, infiltration/seepage and runoff will also be incorporated. Boundary conditions will be defined by the inflow and outflow conditions. An appropriate numerical scheme will be implemented to simulate flow and comparisons with the more detailed St Venant model will be undertaken. This will also be extended to networks.

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WRINKLING OF PAPER LABELS ON BEER BOTTLES

South African Breweries

Industry Representative
Thabo Ntlamelle, South African Breweries, Chamdor, Krugersdorp

Moderator
Astri Sjöberg, University of Johannesburg, Johannesburg

Description

South African Breweries (SAB) Ltd produces thousands of bottles of beer every hour in their factories. The process may be described as follows: Recycled bottles are uncrated, washed and rid of old labels, inspected and rewashed if necessary, filled with beer, sealed, pasteurized, labelled and finally crated before distribution to retailers. The whole process is fully automated. After consumption, the empty bottles are returned to SAB Ltd for recycling, sometimes less than a day after they were produced. SAB Ltd asked the group to study the difficulties they encounter with paper labels. A part of this problem has already been considered at a previous study group in Australia for wine bottles and the present group used their report [1,2] as a starting point.

Executive summary

In the labelling process glue is applied to the paper label by a pallet in thin strips. A sponge presses the centre of the label onto the bottle and a series of brushes flatten the sides of the label. This process may lead to two types of defects:

1. Parts of the label may not adhere to the bottle
2. Long horizontal wrinkles may appear between the glue strips.

Although these defects do not jeopardise the quality of the beer, they may be hugely detrimental to the marketing of the product.

The group focused on three aspects of this wide ranging problem:

- *Optimal speed for the labelling process.*

The first defect is caused by gaps in the adhesion of the label to the bottle. These gaps appear between the region where the label is applied to the bottle by the sponge and the regions which are brushed onto the bottle. The gaps can vary in size, depending on the speed at which the bottles move past the brushes as well as the condition and properties of the brushes. The group derived an optimal production velocity as a function of the geometry of the bottles and the properties of the brushes to minimise the gap. The geometry also gives the largest label size that can be accommodated by the current setup. The gaps could further be reduced and the label size extended by turning the bottles slightly towards either side and sending them through a set of brushes after each turn.

- *Geometry of the glue strips.*

The geometry of the glue strips is a key element in the wrinkling effect. Wrinkles are due to the expansion of the paper in moist conditions in its preferential direction between the glue strips. The group evaluated the importance of the distance between the glue strips. This spacing is a function of the initial glue distribution before the label is glued on the bottle, and also varies with the pressure applied by the brushes. In order to reduce the wrinkles, the group also investigated the influence of the angle between the preferential expansion direction of the paper and the glue strips. The following suggestions concerning the application of the glue could have a drastic impact on the reduction of the occurrence of wrinkles [1]:

- Glue should continue to be applied in glue strips. The spaces between the strips promote drying of the glue and they allow compressed air to escape, thereby avoiding "wallpapering bubbles".

- Glue strips should be perpendicular (or transverse) to the fibers to reinforce the paper against expansion. The fibers in the labels are horizontal and therefore the glue strips should run vertically.

- *Removal of the labels.*

At the start of the washing process, the empty bottles are put in a bath of water and caustic soda for a few minutes. When in contact with the liquid mixture, the glue softens and labels detach from the bottles. The group studied the different processes leading to this removal and assessed the consequences of changing the glue patterns. Water diffusion through the label appears to be the leading process. A modification of the glue distribution should therefore not affect the label removal drastically.

References

[1] Broadbridge, P. Fulford, G.R., Fowkes, N.D., Chan, D.Y.C. and Lassig, C. Bubbles in wet, gummed wine lables, SIAM Review, **41**, (1999), 363-372.

[2] Hewitt, J., Editor, Wet gum labelling of wine bottles. Proceedings of the 1996 Mathematics in Industry Study Group, University of South Australia, Adelaide, 103-113.