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Technical Reports

Mathematical simulation of a glass furnace I: The batch melting of glass

Mathematical simulation of a glass furnace II: Gas diffusion and bubble growth

Predicting glass ribbon shape in the tin bath

Optimal management strategy for white rhinoceros

Optimal flat glass shapes

PREFACE

The tenth Mathematics in Industry Study Group (MISG) in South Africa was held in the School of Computational and Applied Mathematics at the University of the Witwatersrand, Johannesburg, from Monday 14 January to Friday 18 January 2013.

The total number of registered participants at the MISG was sixty-seven. There were twelve academic staff, forty-three graduate students, two postdoctoral fellows, five industry representatives and five invited guests. The invited guests were:

Neville Fowkes	University of Western Australia
Alexander Guterman	Moscow State University, Russia
Mubanga Lombe	University of Zambia, Zambia
Colin Please	Oxford Centre for Collaborative Applied Mathematics, University of Oxford, England
Satyajet Roy	Indian Institute of Technology, Madras, India

The South African Universities and Institutes which were represented were:

University of the Witwatersrand, Johannesburg
African Institute for Mathematical Sciences
University of KwaZulu-Natal
North-West University
Vaal University of Technology

One graduate student participated from:

Institute of Technology, Bandung, Indonesia

The MISG Workshop was opened by Professor Helder Marques, Assistant Dean Research at the University of the Witwatersrand.

The MISG Workshop followed the established format for Study Group meetings held throughout the world. South African industry had been approached to submit problems during 2012. Five problems were submitted. On Monday morning each Industry Representative made a twenty-five minute presentation in which the problem was described and outlined. The academics and graduate students then split into small study groups and worked on the problems of their choice. Some participants worked on one problem while others moved between problems and made contributions to several problems. Each problem was co-ordinated by a senior moderator and one or more student moderators. The role of the senior moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderators was to present short reports at the end of each working day on the progress made that day. The moderators were in contact with the Industry Representatives throughout the meeting. On Friday morning there was a full report back session to industry. Each senior moderator, with assistance from the student moderators, made a twenty-five minute presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes in which to make comments on the progress and results which were reported. The MISG ended at lunch time on Friday.

The MISG was preceded by a Graduate Workshop from Wednesday 9 January to Saturday 12 January 2012. The objective of the graduate Workshop was to provide the graduate students with the necessary background to make a positive contribution to the MISG the following week. The students were given hands-on experience at working in small groups on problems of industrial origin, some of which were presented at previous MISG meetings, at interacting scientifically and at presenting oral reports on their findings. The Facilitator of the Graduate Workshop was Professor Neville Fowkes of the School of Mathematics and Statistics, University of Western Australia. He was assisted by Professor Montaz Ali and Professor Norman Owen-Smith from the University of the Witwatersrand, by Dipo Aldila who is a PhD student from the Institute of Technology in Bandung, Indonesia and by Dario Fanucchi, Gideon Fareo, Ashleigh Hutchinson, Rahab Kgatle and Michael Mitchley who are Higher Degree students at the University of the Witwatersrand. Thirty-eight graduate students participated in the Workshop. Four problems were presented to the graduate students:

Management of rhino removals to maximise the reproductive potential of the rhino population

Reverse flow reactor for greenhouse gas conversion
Design of an ultrafiltration unit
The quadratic assignment problem

The graduate students worked in small study groups on the problem of their choice. Each group presented their results at a report back session on Saturday afternoon.

The sponsors of the Graduate Workshop and the MISG were:

- Hermann Ohlthaver Trust
- African Institute for Mathematical Sciences
- Dean’s Discretionary Fund, Faculty of Science, University of the Witwatersrand
- Mathematical Sciences, University of the Witwatersrand

We thank the sponsors without whose support the Graduate Workshop and the MISG could not have taken place.

Liste of Delegates

A. Academic	
Ali, Montaz	University of the Witwatersrand
Fanucchi, Dario	University of the Witwatersrand
Fareo, Gideon	University of the Witwatersrand
Fowkes, Neville	University of Western Australia, Australia
Guterman, Alexander	Moscow State University, Russia
Harley, Charis	University of the Witwatersrand
Jacobs, Byron	University of the Witwatersrand
Khalique, Masood	North-West University
Lombe, Mubanga	University of Zambia, Zambia
Mason, David	University of the Witwatersrand
Mitchley, Michael	University of the Witwatersrand
Moitsheki, Joel	University of the Witwatersrand
Mubanga, Lombe	University of Zambia, Zambia
Oliphant, Terry-Leigh	University of the Witwatersrand
Owen-Smith, Norman	University of the Witwatersrand
Please, Colin	University of Oxford, United Kingdom
Roy, Satyajit	Indian Institute of Technology Madras, India
B. Postdoctoral Fellows	
Sawyerr, Babatunda	University of Kwa-Zulu-Natal
Suriyakat, Wannaporn	University of the Witwatersrand
C. Graduate Students	
Agbavon, Koffi	African Institute for Mathematical Sciences
Aldila, Dipo	Institute of Technology in Bandung, Bandung, Indonesia
Boloka, Moloko	University of the Witwatersrand
Brown, Stephanie	University of the Witwatersrand
Carrim, Abdul	University of the Witwatersrand
Comu, Campher	University of the Witwatersrand
Dolamo, Morepe	University of the Witwatersrand

Earle, Adam	University of the Witwatersrand
Gidey, Hagos	African Institute for Mathematical Sciences
Hutchinson, Ashleigh	University of the Witwatersrand
Jacobs, Kyle	University of the Witwatersrand
Joel, Luke	University of KwaZulu-Natal
Keebine, Zandile	University of the Witwatersrand
Kekana, Palesa	University of the Witwatersrand
Kgatle, Rahab	University of the Witwatersrand
Knights, Michelle	University of Cape Town and AIMS
Kokela, Lady	University of the Witwatersrand
Lalla, Vimal	University of the Witwatersrand
Louw, Kirsten	University of the Witwatersrand
Maduna, Nhlanhla	University of the Witwatersrand
Magalakwe, Gabriel	North-West University
Magan, Avnish	University of the Witwatersrand
Makhalemele, Cynthia	Vaal University of Technology
Mamba, Siphamandla	University of the Witwatersrand
Mbongo, Jeffrey	African Institute for Mathematical Science
Mindu, Nkululeko	University of the Witwatersrand
Mogole, Salome	University of the Witwatersrand
Mohale, Mantshokhi	University of the Witwatersrand
Mohlala, Selekeng	University of the Witwatersrand
Mothlele, Tshepiso	University of the Witwatersrand
Mponda, Francis	African Institute for Mathematical Sciences
Munthali, Richard	African Institute for Mathematical Sciences
Newby, Eric	University of the Witwatersrand
Nyandeni, Zamashobane	University of the Witwatersrand
Phahladira, Lee	University of the Witwatersrand
Phillips, Raymond	University of the Witwatersrand
Ramushu, Boledi	University of the Witwatersrand
Raphulu, Dzanga	University of the Witwatersrand
Sangweni, Zinhle	University of the Witwatersrand

Simelane, Siaybonga	University of the Witwatersrand
Soko, John	University of the Witwatersrand
Tuyishimire, Emmanuel	African Institute for Mathematical Sciences
Woolway, Matthew	University of the Witwatersrand
D. Industry Representatives	
Ferreira, Eddie	PFG Glass, Springs
Lemako, Lemako	Acting Director Corporate Services, Dihlabeng Local Municipality, Free State
Mugumbate, Joseph	Mathematics Teacher, Waverley Girls High School, Johannesburg
't Sas-Rolfes, Michael	Conservation Economist (Independent)
Von Wielligh, Riaan	PFG Glass, Springs

PROBLEMS

MATHEMATICAL SIMULATION OF A GLASS FURNACE

Industry: Glass

Industry Representative

- Eddie Ferreira, PFG Glass, Springs

Problem Statement

To improve and control a process optimally, it is necessary to understand and measure the important parameters. This is no different for a glass melting furnace. The chemistry and physics is complex and interdependent. Furthermore, the very high temperatures employed make it very difficult to measure and see everything that is important. However, by making certain assumptions and simplifying it is possible to simulate the process using mathematical models. The accuracy of the simulation depends on the assumptions made and the models used.

It is fairly simple to model the glass flow using the three conservative laws:

- Continuity:
- Momentum – conservation
- Energy conservation:

The general purpose CFD code Ansys Fluent® is implemented at PFG Building Glass to model the glass flows and the combustion space which solves the conservation equations and selected physical models.

The steps involved in solving a CDF problem are:

1. Define the modelling goals
2. Create the model geometry and mesh
3. Set up the solver and physical models
4. Compute and monitor the solution

5. Examine and save the results
6. Consider revisions to the numerical model parameters if necessary

The study group is asked to implement sub-models in step 3 above for the following two processes:

- The batch melting of glass
- The fining and refining process

The end goal is for the models to be implemented and tested in C or Ansys Fluent®.

MODELING GOALS

1. Model the batch melting process
    a. Options
    b. Inputs: empirical constants defining chemical reaction rates, specie properties
    c. Physical models: reaction rates, specie diffusivity, source terms
    d. Assumptions
    e. Computational domain boundaries
    f. Boundary conditions
    g. Mesh resolution

During the melting process, a blanket of foam (batch) made up of various chemical by-products forms on top of the molten glass (melt). The batch blanket is constantly melting into the melt. The task is to model the formation and evolution of this batch blanket and its interaction with the melt.

2. Model the fining and refining process
    a. Options
    b. Inputs: empirical constants defining chemical reaction rates, specie properties
    c. Physical models: gas diffusion and bubble growth
    d. Assumptions

e. Computational domain boundaries
f. Boundary conditions
g. Mesh resolution

During the melting process after the batch has melted, the temperature gets high enough for fining gases to be created by chemical reactions. These gases diffuse through the melt into existing bubbles in the melt. The task is to model this diffusion process and the evolution of these bubbles and their dependence and interdependence on temperature.

PREDICTING GLASS RIBBON SHAPE IN THE TIN BATH

Industry: Glass

Industry Representative

- Eddie Ferreira, PFG Glass, Springs

Problem Statement

During the forming process, the glass-melt flows on top of a bath of molten tin. The forming of glass is essential to the final dimensions of the glass ribbon. The interaction of the bath (tin flows, heating, cooling and forming equipment) with the ribbon is complex and bidirectional, that is, the ribbon shape can affect the bath flow and vice versa. The final dimension of the ribbon is set by customer requirements. The final glass ribbon can suffer optical distortion related to the forming of the ribbon in the tin bath, which impacts negatively on the quality of the glass product.

A model is required that will predict the ribbon shape in the bath based on typical operational parameters. This will help determine better operating parameters for prevailing conditions in the bath or to trouble shoot optical distortion.

Modelling Goals

- The differential equation describing the ribbon deformation
- The method of supplying the boundary conditions.
- A method of solving for the ribbon shape given operational parameters.
- A method of coupling the ribbon shape with commercial CFD solvers.

OPTIMAL MANAGEMENT STRATEGY FOR WHITE RINOCEROS

Industry/sector: Conservation/game ranching

Industry Representative

- Michael ’t Sas-Rolfes, Conservation Economist (Independent)

Problem Statement

South Africa’s white rhino population is under increasing threat from illegal killing (poaching) to supply the demand for rhino horn in East Asia. As a response to this threat, some rhino owners are taking defensive measures such as dehorning live animals and adopting intensive management practices (elevated stocking rates with supplementary feeding, sometimes outside the rhinos’ natural habitat or traditional range).

Conservationists and certain other interest groups (eco-tourists, trophy hunters) wish to ensure the continued survival of the extensive, managed (wild) white rhino population, that is not subject to such interventions, that is. rhinos are not dehorned, overstocked, artificially fed or otherwise genetically manipulated. However, to maintain rhinos in such extensive conditions is more costly (especially in terms of increasing security needs).

At this time trade in rhino horn is illegal, both within South Africa and to international markets. Some rhino owners argue that re-establishing a legal trade could provide an additional source of income to offset protection costs, thereby helping with rhino conservation. However, such a move would most likely benefit intensive owner/managers more than extensive owner/managers.

Given various production and cost parameters, we can determine the likely effect of a change in trade policy and various demand parameters (as reflected by the market price of rhino horn). If rhino specialist can specify a minimum viable population of rhinos to retain under extensive conditions, we can also determine whether and when a legal trading regime might threaten the so-called wild populations and we can also determine whether intensive managers may need to subsidize the protection costs borne by extensive managers and to what extent.

Modelling goals

- Model the production functions of three types of producers: intensive, extensive and illegal to determine optimal harvesting levels (this is an optimal control problem)
- Model the interaction between the three types of producers under two different strategies: legal trade and illegal trade
- Model the effects of changing demand conditions (using prices as a proxy)

Intensive production function
Cost: Fences, feedstock, security, dehorning
Revenues: Live sales, horn sales (under legal trade)

Extensive production function
Costs: Fences, security, veterinary
Revenues: Conservation subsidies, tourism, trophy hunting, live sales

Illegal harvest function
Costs: equipment, transport, bribes, probable cost of punishment
Revenue: Horn sales

OPTIMAL FLAT GLASS SHAPES

Industry: Glass

Industry Representative:

- Riaan von Wielligh, PFG Glass, Springs

Problem Statement

In the glass Industry our group manufactures and caters for various segments of the South African and International markets in terms of glass quality and application.

In the Domestic Automotive stream we manufacture glass from sand and supply finished glass components to the Local Motor manufacturers, the replacement glass for vehicles on the road as well as replacement glass for vehicles in Europe.

As manufacturer of glass we receive forecasts from Manufacturers and Distributors of glass which must be consolidated and converted to manufacturing requirements. Because of unpredictability of the requirements and the wide range of products that we offer as a group we need to rationalise the range of flat sizes that we produce as input to produce the final shapes in our Automotive Glass manufacturing division.

An optimisation model is required where the group can enter our flat glass forecast requirements and the model will calculate the optimal flat glass shapes that we need to manufacture in anticipation of the actual orders received.

Real data will be provided to solve the problem, it will be a 2D optimisation problem where the input will be provided in square or rectangular shapes and the output must be provided as a manufacturing requirement in square or rectangular shapes, taking into consideration the manufacturing constraints of the process.

Modelling Goals

To minimise the glass wastage from the manufacturing process to the final shape produced for our customer base. (The cutting up of a larger shape to a smaller final shape.)

The model must take the various manufacturing parameters of the production lines that produce the required glass shapes into consideration.

There are also product attributes that must be taken into consideration: colour, thickness and orientation.

At the end of the day a model is required to convert a Forecasted requirement into an optimal Manufacturing requirement, taking into consideration the physical process constraints.

Technical Reports