

**PROCEEDINGS OF THE  
MATHEMATICS IN INDUSTRY  
STUDY GROUP**

**2012**

## **Mathematics in Industry Study Group South Africa MISGSA 2012**

The manuscripts for the Proceedings of the MISGSA were written by the problem moderators in consultation with the other members of the study group for that problem and the industry representative.

The Editors of the Proceedings were

Prof D P Mason (University of the Witwatersrand, Johannesburg, South Africa)  
Prof N D Fowkes (University of Western Australia, Australia)

The Technical Reports were submitted to the Editors. Each Report was refereed by one referee. On the recommendation of the referees the Reports were accepted for the Proceedings subject to corrections and minor revisions. The Editors would like to thank the referees for their assistance by refereeing the Reports for the Proceedings.

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## PREFACE

The ninth Mathematics in Industry Study Group (MISG) in South Africa was held in the African Institute for Mathematical Sciences, Muizenberg, Cape Town, from Monday 9 January to Friday 13 January 2012.

There were fifty-nine participants at the MISG. Six academic staff, thirty-eight graduate students, seven industrial representatives and eight overseas guests attended. The overseas guests were:

Chris Beward	Oxford Centre for Collaborative Applied Mathematics (OCCAM), Oxford University, England
Neville Fowkes	University of Western Australia, Australia
Graeme Hocking	Murdoch University, Western Australia, Australia
Kamil Kulesza	Centre for Industrial Applications of Mathematics and Systems, Warsaw, Poland
Sarah Mitchell	University of Limerick, Ireland
Tim Myers	Centre de Recerca Matemàtica, Barcelona, Spain
Colin Please	University of Southampton, England

The South African Universities and Institutes which were represented were:

African Institute for Mathematical Sciences  
University of the Witwatersrand  
University of Pretoria  
North-West University  
University of KwaZulu-Natal  
UNISA

The MISG Workshop was opened by Professor Barry Green, Director of the African Institute for Mathematical Sciences (AIMS)

The MISG Workshop followed the established format for Study Group meetings held throughout the world. South African industry had been approached to submit problems during 2011. Eight problems were submitted. On Monday morning each Industry Representative made a twenty-five minute presentation in which the problem was described and outlined. The academics and graduate students then split into small study groups and worked on the problems of their choice. Some participants worked on one problem while others moved between problems and made contributions to several problems. Each problem was co-ordinated by a senior moderator and one or more student moderators. The role of the senior moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderators was to present short reports at the end of each working day on the progress made that day. The moderators were in contact with the Industry Representatives throughout the meeting. On Friday morning there was a full report back session to industry. Each senior moderator, with assistance from the student moderators, made a twenty-five minute presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes in which to make comments on the progress and results which were reported. Work was done on only seven of the eight problems submitted. There were not sufficient participants knowledgeable in control theory and optimization to work on the eighth problem. The MISG ended at lunch time on Friday.

The MISG was preceded by a Graduate Workshop from Wednesday 4 January to Saturday 7 January 2012. The objective of the graduate Workshop was to provide the graduate students with the necessary background to make a positive contribution to the MISG the following week. The students were given hands-on experience at working in small groups on problems of industrial origin, some of which were presented at previous MISG meetings, at interacting scientifically and at presenting oral reports on their findings. The Facilitator of the Graduate Workshop was Professor Neville Fowkes of the School of Mathematics and Statistics, University of Western Australia. He was assisted by Brendan Florio, a PhD student at the University of Western Australia and by Daris Fanucchi and Eric Newby, Masters students at the University of the Witwatersrand. Twenty-seven graduate students participated in the Workshop. Four problems were presented to the graduate students:

Geothermal heat extraction  
Fishing pole design

Thermal unit commitment problem  
Building a multi-agent intelligent controller

The graduate students worked in small study groups on the problem of their choice. Each group presented their results at a report back session on Saturday afternoon.

**The sponsors of the Graduate Workshop and the MISG were:**

- Hermann Ohlthaver Trust
- African Institute for Mathematical Sciences
- Oxford Centre for Collaborative Applied Mathematics (OCCAM)
- Centre de Recerca Matemàtica, Barcelona, Spain
- Dean's Discretionary Fund, Faculty of Science, University of the Witwatersrand
- Professor Yunus Ballim, Deputy Vice-Chancellor (Academic), University of the Witwatersrand
- School of Computational and Applied Mathematics, University of the Witwatersrand

We thank the sponsors without whose support the Graduate Workshop and the MISG could not have taken place.

## **LIST OF DELEGATES**

### **Academic**

Adewumi, Aderemi	University of KwaZulu-Natal
Ali, Montaz	University of the Witwatersrand
Breward, Chris	Oxford Centre for Collaborative Applied Mathematics, Oxford University, England
Fowkes, Neville	University of Western Australia, Australia.
Harley, Charis	University of the Witwatersrand
Hocking, Graeme	Murdoch University, Perth, Australia
Kulesza, Kamil	Centre for Industrial Applications of Mathematics and Systems, Engineering Polish Academy of Science, Warsaw, Poland
Khalique, Masood	North-West University
Mason, David	University of the Witwatersrand
Mitchell, Sarah	University of Limerick, Ireland
Moitsheki, Joel	University of the Witwatersrand
Myers, Tim	Centre de Recerca Matematica, Barcelona, Spain
Ochendon, Hilary	Oxford Centre for Industrial and Applied Mathematics, Oxford University, England
Please, Colin	University of Southampton, England

### **Graduate Students**

Akinlotan, Deborah Morenikeji	African Institute for Mathematical Sciences
Amikya, Emmanuel	African Institute for Mathematical Sciences
Ajibola, Ariyo	University of KwaZulu-Natal
De Decker, Michelle	Centre de Recerca Matematica, Barcelona, Spain
Earle, Adam	University of the Witwatersrand



Fanucchi, Dario	University of the Witwatersrand
Fareo, Gideon	University of the Witwatersrand
Florio, Brendan	University of Western Australia, Australia
Fodya, Charles	University of the Witwatersrand
Hamese, Maudu	University of the Witwatersrand
Hutchenson, Ashleigh	University of the Witwatersrand
Ikpe, Dennis Chinemerem	University of South Africa, Pretoria
Jacobs, Byron	University of the Witwatersrand
Kgatke, Rahab	University of the Witwatersrand
Kika, Jesal	University of the Witwatersrand
Kokela, Lady	University of the Witwatersrand
Lucas, Luke	University of KwaZulu-Natal
Martins, Arasomwan	University of Kwa-Zulu-Natal
Mindu, Nkululeko	University of the Witwatersrand
Mogole, Salome	University of the Witwatersrand
Mohamed, Ahmed	African Institute for Mathematical Science
Ndlovu, Luyanda	University of the Witwatersrand
Nengome, Mulalo	University of the Witwatersrand
Ngomane, Derwin	University of the Witwatersrand
Newby, Eric	University of the Witwatersrand
Obagbuwa, Christiana	University of KwaZulu-Natal
Olusanya, Michael	University of KwaZulu-Natal
Padayachi, Nadine	University of the Witwatersrand
Philips, Raymond	University of the Witwatersrand
Potsane, Moshe	University of the Witwatersrand
Rademeyer, Maryke	University of the Witwatersrand

Randriafanomezantsoa, George	African Institute for Mathematical Science
Rusagara, Innocent	University of the Witwatersrand
Sangweni, Zinhle	University of the Witwatersrand
Sejeso, Matthews	University of the Witwatersrand
Simelane, Simphiwe	University of the Witwatersrand
Suleiman, Aminat	African Institute for Mathematical Science
Taylor, Asha	University of the Witwatersrand

### **Industry Representatives**

Adewumi, Aderemi	School of Computer Science, University of KwaZulu-Natal
Attridge, Sandy	Physiotherapist, University of the Witwatersrand
Burke, Michael	Mobile Intelligent Autonomous Systems, CSIR, Pretoria
Loubser, Richard	Sugar Milling Research Institute, Durban
M <sup>o</sup> Donald, Brendon	Customs and Tax Management Solutions, Cape Town
Pedro, Jimoh	School of Mechanical Engineering, University of the Witwatersrand
Sabatta, Deon	Mobile Intelligent Autonomous Systems, CSIR, Pretoria
Yilmaz, Halil	School of Mining Engineering, University of the Witwatersrand

# PROBLEMS

# RISING WATER TABLE AND SEISMICITY

**Industry:** Mining

## **Industry Representatives**

- Nielen van der Merwe, School of Mining Engineering, University of the Witwatersrand, Johannesburg
- Halil Yilmaz, School of Mining Engineering, University of the Witwatersrand, Johannesburg

## **Problem Description**

The old now closed gold mines in the Witwatersrand are in the process of flooding. While most of the attention is on the environmental aspect with the possibility of the contaminated water decanting out of the mines, there is another, lesser known aspect that deserves attention.

The mine workings are intersected by numerous geological faults that are stable by virtue of the frictional resistance to movement and the clamping effect of stresses normal to the faults plane. It is possible that the rising water table, resulting in elevated levels of water pressure at depth, will cause water in the deeper areas of the mines to ingress into the fault planes. This will reduce the frictional resistance to movement across the faults as well as the effective resistance to movement across the faults as well as the effective normal stress to the fault plane, increasing the likelihood of movement. If the faults move, there will be seismic events (small to medium earthquakes).

If this happens, a city like Johannesburg could experience small earthquakes. There is at least a suspicion that it may already have happened on a small scale.

The challenge is to quantify the relationship between water pressure and the frictional resistance to movement across the fault planes so that the likelihood of seismic events can be estimated.

# LIQUID FLOW THROUGH SHREDDED CANE

**Industry:** Sugar

## Industry Representative

- Richard Loubser, Sugar Milling Research Institute, University of KwaZulu-Natal

## Problem Description

In South Africa and some other countries, sugar is extracted using a device known as a diffuser similar to the schematic in Figure 1. Shredded cane is introduced at the front end of the diffuser and fresh water at the back. The water percolates through the bed and is collected in trays below. The water is then pumped to the top of the bed where once again it percolates through the bed. This process is repeated until the water, having washed out the sucrose from the cane reaches the front of the diffuser. The cane is moved steadily towards the back of the diffuser. On the way the concentration of sucrose in the cane reduces. This gives rise to a counter current extraction process.

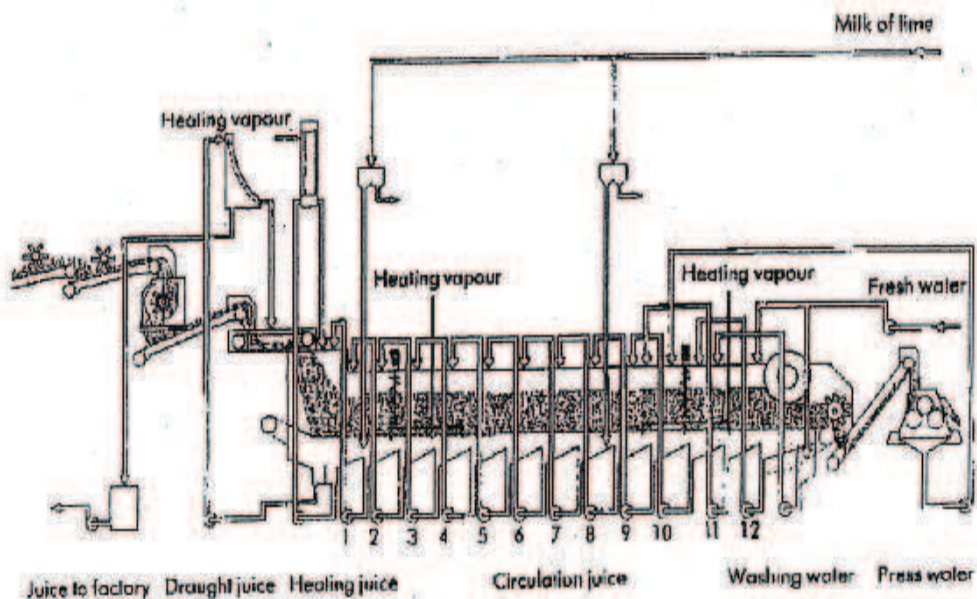


Figure 1. Diffuser schematic

The ideal situation would be that the juice percolates through as a plug and emerges in a single outlet trough. This, however, is not the case and tracer tests have shown that juice from a single injection point can spread over as many as five outlet cells when it reaches the bottom of the cane bed, leading to inefficiencies in the device.

Shredded cane has two major components:

1. A fibre portion ( $\pm 70\%$  of the cane by dry mass) consisting of slender pieces between 25 and 100 mm long and between 1 and 5 mm thick. The randomly orientated fibres form a bed.
2. The remainder ( $\pm 30\%$  of the cane by dry mass) is fine material made up of pith and short fibres.

As the juice flows vertically through the bed it may encounter various restrictions.

These include:

- i. Accumulation of the pith at the bottom of the bed.
- ii. Air filled voids, up to 120 mm diameter, through which no liquid flow occurs.
- iii. The randomly orientated fibres.

A typical diffuser:

1. Has a length of about 60m
2. Would have 14 stages, increasing in length to account for reduction in percolation rate.
3. Has a bed height of 1.2 m.
4. Has a bed travel velocity of about 0.8 m/min.

The objectives of the study are to investigate:

1. Validity of studies carried out in columns to simulate a section of the diffuser.
  - Will the effect of the walls on the lateral flow component have a detrimental effect on the quality of the measurements taken?
2. Which cane characteristics are desirable for optimal extraction in a diffuser?
  - Are long fibre lengths desirable because of the tendency to form voids?
  - How much influence does pith-in-cane and short fibres have?

3. What can be done to foster plug flow?

- Alternatively, what is the cause of the very wide dispersion of the juice away from plug flow? Is it due to the voids, the fibrous nature of the bed, random effects from flow in a packed bed, or all of the above?

# LIQUID FLOW IN A SUGAR CENTRIFUGAL

**Industry:** Sugar

## **Industry Representative**

- Richard Loubser, Sugar Milling Research Institute, University of KwaZulu-Natal

## **Problem Description**

In the process of manufacturing sugar, the concentration of the sugar in the extracted juice is increased by evaporation. Further water is evaporated in a vacuum pan to the point of supersaturation. Seed crystals are added to the mix which is cooled allowing the crystal to grow. The product that results contains sugar crystals (say 50% by mass) and mother liquor with an average viscosity of roughly 0.8 Pa.s (the mother liquor is actually a non-Newtonian fluid). A centrifugal is used to separate the sugar crystal from the mother liquor.

The centrifugal consists of a cylindrical basket. The basket has holes through which the liquid (molasses) can pass. A fine mesh liner is used to contain the crystals.

When massecuite is loaded into the basket the speed is 200 rev/min. A typical centrifugal has a diameter of 1.2 m and rotates at 960 rev/min to separate the molasses from the crystal. The massecuite forms a layer of about 100 mm thick on the basket during this process. During the separation part of the cycle, water may be sprayed onto the crystal to aid in the removal of more impurities, Steam is used to heat the mixture to aid the flow of the liquor through the crystal layer. The basket is again slowed down to allow a plough arrangement to remove the sugar crystal.

The colour of the sugar is controlled by the amount of water added to the process, which washes off residual mother liquor (molasses) from the surface of the crystals. The water does, however, dissolve some of the sugar leading to losses. It may also be possible to reduce the colour by adding oxidising agents such as peroxide to the wash water.



If the flow through to crystal layer is obstructed, this can result in the formation of a supernatant liquid layer within the cylindrical basket. The system can then become dynamically unstable with catastrophic results.

The aim of the study is to investigate the relationships between basket speed, water addition, recovery efficiency, colour removal, the effectiveness of added chemicals and safe operation of the centrifugal.

Specifically, it would be desirable to be able to model the flow of (1) molasses and (2) wash water through the bed of crystals under the applied centrifugal force to determine the flow rates as a function of liquid properties and crystal size distribution (to be supplied), with a view to determining:

1. Under what conditions flow is impeded such that a supernatant layer is formed and under what conditions this will lead to dangerous dynamic instability.
2. The effectiveness of the displacement of the residual molasses film by water to endure maximum “washing” effect with minimum dissolution of the crystals. In this respect, channeling of the wash water is something observed whereby the water selectively flows through part of the bed and does not wash the remainder.
3. The optimum loading conditions for the mixture of crystal and mother liquor, which has to flow up the wall of the basket to make an even layer before the machine is sped up. If the loading speed is too high, the molasses drains out before the load has spread out evenly, leading to a non-uniform bed and consequent non-uniform washing.

# SWIM-FIN PARAMETERS AND THEIR RELATIONSHIP TO INJURY

**Industry:** Sport, stillwater life saving

**Industry Representative:**

- Sandy Attridge, Physiotherapist

## Problem Description

Still water life saving is a pool based sport with competition at an international level. A specialized fin of large size is used in certain events. Competition rules regulate the fin length (regardless of the athletes size), but the competitor can choose to change the width of the fin within certain limits, as well as the stiffness. Sometimes the same athlete may use different fins for various events.

The general trend within the sport is to choose the widest and stiffest fin. Injuries have been reported in the ankle, knee, hip joints as well as lower back of the athletes using these swim fins.

Task:

1. Investigate the efficiencies of the various combinations of width and stiffness of the swim fin used by the Stillwater Lifesavers.
2. Investigate the forces generated by the use of the various fin parameter combinations on the different joints / points along the kinetic chain,

It is hoped that this may help with identification of injury patterns and serve to assist in lowering injury in these athletes.

# TRACKING AUTONOMOUS AGENTS WITH COMPLEX BEHAVIOUR

**Industry:** Robotics

## Industry Representatives

- Deon Sabatta, Mobile Intelligent Autonomous Systems, CSIR, Pretoria
- Michael Burke, Mobile Intelligent Autonomous Systems, CSIR, Pretoria

## Problem Description

Autonomous robots perform actions based on a simplified internal representation of their environment. The fidelity of these “world models” directly affects their ability to respond correctly to changes in the environment. For static environments, the creation of a world model is trivial; however, the introduction of other autonomous agents complicates this process. Other autonomous agents typically react to the world in a deterministic manner. The goal of this problem is therefore to create a general tracking and prediction algorithm capable of forecasting the movement and behaviour of such an autonomous agent.

The solution to the tracking problem will be evaluated within the Robocode programming framework (<http://robocode.sourceforge.net/>). Robocode is a programming game, where the goal is to develop an autonomous tank that battles against other tanks. Typically, no knowledge of the behaviour of competitors is available and this behaviour needs to be learnt in the game. The Robocode framework provides a valuable means of model verification, as the accuracy of a motion model can be evaluated by considering the hit rate when firing at target positions predicted by the model.

The Robocode framework requires limited programming knowledge so is particularly suited to the rapid evaluation of mathematical algorithms. Participants will be able to evaluate their contributions and modelling attempts by competing against sample agents and each other within the Robocode environment.

A solution to the problem of tracking autonomous agents with complex behaviour is of particular interest in field robotics. Field robotics refers to the deployment of intelligent robots in dynamic and unstructured environments. Here, robots frequently require knowledge of the motion of surrounding objects to enable path planning in dynamic and cluttered environments. As a result, algorithms allowing for the successful tracking of pedestrians, vehicles and other autonomous agents are crucial to the success of these robotic systems.

# MULTIVARIABLE AND MULTIOBJECTIVE CONTROLLER DESIGN AND OPTIMIZATION PROBLEM FOR ACTIVE VEHICLE SUSPENSION SYSTEMS

**Industrial:** Automobile

## Industry Representative

- Jimoh Pedro, School of Mechanical Engineering, University of the Witwatersrand

## Problem Description

Vehicle suspension is meant to simultaneously isolate the vehicle body and occupants from terrain induced vibration (rich comfort), provide good contact between the road and the vehicle tyres (road holding), ensure good handling and enhance braking. To achieve this, an active vehicle suspension system is required. This calls for the addition of an actuator to the suspension system and the actuator needs a controller to regulate the forces that the actuator sets up between the wheel and the body of the vehicle.

The nonlinear nature of the vehicle suspension problem makes the application of computational intelligence techniques like neural network, fuzzy logic and evolutionary algorithms readily attractive for its control, especially when popular modern control methods like  $H_2$  and  $H_\infty$  often result in high order problems that are difficult to solve. The establishment of system stability is relatively more challenging when applying computational intelligence based control techniques.

In spite of the drawbacks of the PID control technique in terms of robustness to parameter variations and system uncertainties, PID-based control remains the most popular control technique in terms of industrial application. This is due to its simplicity and ease of tuning. Several amendments have been made to the classical PID control design to enhance its performance in terms of robustness, but these come with additional gains to be tuned.

A possible solution could lie in combining computational intelligence based control methods with PID. The effectiveness of PID control means being able to obtain controller gains that enable the controller to meet the following requirements:

1. Nominal system stability
2. Good command following
3. Performance within the various design specifications.

Applying a force-balance analysis to the full-car model gives the governing system of equations for the heave motion, the pitch motion, the roll motion and the vertical motions at the four tires [1]. The challenges that could be encountered include the following:

1. The system of equations is a seven degree-of-freedom problem with inter-coupling of the dynamic modes.
2. Conflicting design objectives requiring trade-offs
3. Multi-objective and multivariable optimization
4. Robustness to system nonlinearities, parameter variation and uncertainties.
5. Low sensitivity to variation in road disturbance input.

Task:

1. Design a suitable PID or PID based controller suitable for this seven degree-of-freedom problem.
2. Develop an optimization algorithm to select the optimal gains for the designed controller

The optimization algorithm developed must be able to enable:

- i. The optimal gains to be tuned online, and
- ii. Robustness to parameter variation and uncertainties

## References

1. P. Gasper, Z, Szabo and J. Bokor. Active Suspension in Integrated Vehicle Control. In, Switched Systems (ed. Janusz Kleban), Intech Open Access Publishers, 2009. Available at: [http://www.intechopen.com/source/pdfs/6792/Intech-Active\\_suspension\\_in\\_integrated\\_vehicle\\_control.pdf](http://www.intechopen.com/source/pdfs/6792/Intech-Active_suspension_in_integrated_vehicle_control.pdf)

# OPTIMIZING AN AGILE-DRIVEN PROJECT SCHEDULE USING SWARM INTELLIGENCE AND SIMULATION TECHNIQUES

**Industry partner:** Tatis-Customs and Tax Management Solutions

## Industry Representatives

- Brendon M<sup>c</sup>Donald, Customs and Tax Management Solutions, Cape Town
- Aderemi Adewumi, School of Computer Science, University of KwaZulu-Natal

## Problem Description

Project scheduling is one of the most intractable optimization problems known. It exists within many domains and has been extensively researched within the last few decades. The standard resource constrained project scheduling problem (RCPSP) is known to be NP-hard in the strong sense and this has often led Project Managers to settle for any robust schedule available, rather than an optimal one. Our problem domain lies specifically in software development project scheduling, where typically the software development environments followed a traditional waterfall approach. Subsequently the development and adoption of the formal agile manifesto has taken place, specifying the best practices of an agile software development environment.

For a project to be successful it needs careful planning, scheduling and control and while agility improves the development cycle it certainly adds complexity to scheduling. Project Managers are having to adapt their scheduling algorithms and methods to best handle the complexities introduced by being agile. From a more technical point of view the schedule needs to incorporate “proactive-reactive” strategies and support preemptive behaviour, while still conforming to the classical RCPSP constraints. In other words the schedule needs to be able to remain stable under a variety of disturbances. If assumptions are violated it should have little consequence and it should satisfy performance requirements predictably in an uncertain environment.

## General RCPSP

The problem  $\mathbf{m}, 1|\mathbf{cpm}|C_{max}$  represents the general non-preemptive RCPSP where  $\mathbf{m}, 1$  are the resource constraints,  $\mathbf{cpm}$  are the precedence constraints and  $C_{max}$  is the objective

function. This can further be modelled conceptually as a linear programming model in the following way:

$$\min f_n \quad (1)$$

subject to

$$f_i \leq f_j - d_j \quad \text{for all } (i, j) \in A \quad (2)$$

$$f_1 = 0 \quad (3)$$

$$\sum_{i \in S_t} r_{ik} \leq a_k \quad \text{for } k = 1, \dots, m \quad \text{and } t = 1, \dots, f_n. \quad (4)$$

In this formulation, the use of a dummy start as well as a dummy end activity is assumed. The decision variables  $f_i$  denote the finish times of the different activities, while the  $d_i$  denotes the duration of each activity,  $a_k$  the availability of the  $k^{th}$  resource type and  $r_{ik}$  the resource requirement of activity  $i$  for the resource type  $k$ . The set  $S_t$  that is used in Eq.4 denotes the set of activities that are in progress at time  $t$ . Assume all resources are renewable.

The objective function Eq.1 minimizes the finish time of the dummy end activity. Eq.2 expresses the precedence relations, while Eq.3 forces the dummy start activity to finish at time 0. Finally, Eq.4 expresses that at no time instant during the project horizon may the resource availability be violated.

Objectives:

1. Using the general RCPSP model above, manipulate and change to best represent a preemptive (activities that can be interrupted), proactive-reactive, stochastic activity timed schedule.
2. Create a mocked project with 20 activities and associated resource and precedence constraints. Each activity needs an associated stochastic activity duration and resource requirement. Include precedence constraints if the activity depends on the completion of other activities before it can commence.
3. Using 1 and 2 above solve for and optimize the respective schedule.
4. Investigate the use of heuristic techniques and compare with results obtained from 3.

# **Technical Reports**