PROCEEDINGS OF THE MATHEMATICS IN INDUSTRY STUDY GROUP

2010

Mathematics in Industry Study Group South Africa
MISGSA 2010

The manuscripts for the Proceedings of the MISGSA were written by the problem moderators in consultation with the other members of the study group for that problem and the industry representative.

The Editor of the Proceedings was

Prof D P Mason (University of the Witwatersrand)

The Detailed Technical Reports were submitted to the Editor. Each Report was refereed by one referee. On the recommendation of the referees the Reports were accepted for the Proceedings subject to corrections and minor revisions. The Editor would like to thank the referees for their assistance by refereeing the Reports for the Proceedings.

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PREFACE

The seventh Mathematics in Industry Study Group (MISG) Workshop in South Africa was held in the African Institute for Mathematical Sciences, Muizenberg, Cape Town, from Monday 11 January to Friday 15 January 2010..

There were forty-nine participants at the MISG. Ten academic staff, three postgraduate fellows, twenty-eight postgraduate students, five industrial representatives and three overseas guests attended. The invited guests were:

Dr Jean Charpin:	University of Limerick, Ireland
Professor Alistair Fitt:	University of Southampton, England
Professor Neville Fowkes:	University of Western Australia, Australia

The South African Universities which were represented were:

African Institute for Mathematical Sciences
University of the Witwatersrand
University of Pretoria
North-West University
University of Cape Town
UNISA
Durban University of Technology

The MISG Workshop was opened by Professor Fritz Hahne, Director of the African Institute for Mathematical Sciences (AIMS).

The MISG Workshop followed the established format for Study Group meetings held throughout the world. South African industry had been approached to submit problems during 2009. Seven problems were submitted. On Monday morning each Industry Representative made a twenty-five minute presentation in which the problem was described and outlined. The academics and graduate students then split into small study groups and worked on the problems of their choice. Some participants worked on one problem while others moved between problems and made contributions to several problems. Each problem was co-ordinated by a senior moderator and a student moderator. The role of the senior moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderator was to present the fifteen minute progress report on Wednesday afternoon. The moderators were in contact with the Industry Representative throughout the meeting. On Friday morning there was a full report back session to industry. Each senior moderator made a twenty-five minute presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes in which to make comments on the progress and results which were reported. The MISG ended at lunch time on Friday.

The MISG was preceded by a Graduate Workshop from Wednesday 6 January to Saturday 9 January 2010. The objective of the graduate Workshop was to provide the graduate students with the necessary background to make a positive contribution to the MISG the following week. The students were given hands-on experience at working in small groups on problems of industrial origin, some of which were presented at previous MISG meetings, at interacting scientifically and at presenting oral reports on their findings. The Facilitator of the Graduate Workshop was Professor Neville Fowkes of the School of Mathematics and Statistics at the University of Western Australia. He was assisted by seven tutors, Dario Fanucchi, Michael Mitchley, Tanya La Foy and Terry Oliphant from the University of the Witwatersrand and Cedrick Kitio, Hermane Mambili-Mamboundou and La Fras Uys from AIMS. A total of twenty-two graduate students participated. Four problems were presented to the graduate students:

Modelling a fishing rod for optimal casting
Making ethanol from grass
Imaging for security
The brewery and the brewing process

The graduate students worked in small study groups on the problem of their choice. Each group presented their results at a report back session on Saturday afternoon.

The sponsors of the MISG were:

African Institute for Mathematical Sciences
Dean's Discretionary Fund, Faculty of Science, University of the Witwatersrand
Oxford Centre for Collaborative Applied Mathematics (OCCAM)

We thank the sponsors without whom the Mathematics in Industry Study Group meeting could not have taken place.

LIST OF DELEGATES

Academic

Ali, Montaz	University of the Witwatersrand
Charpin, Jean	University of Limerick, Ireland.
Fitt, Alistair	University of Southampton, England
Fowkes, Neville	University of Western Australia, Perth, Australia
Hahne, Fritz	African Institute for Mathematical Sciences
Khalique, Masood	North-West University
Laurie, Henri	University of Cape Town
Mason, David	University of the Witwatersrand
Mitchley, Michael	University of the Witwatersrand
Moitsheki, Joel	University of the Witwatersrand
Mureithi, Eunice	University of Pretoria
Robin, Amanda	University of the Witwatersrand
Sears, Michael	University of the Witwatersrand

Postdoctoral Fellows

Kwuimy Kitio, Cedric	African Institute for Mathematical Sciences
Mambili-Mamboundou, Hermane	African Institute for Mathematical Sciences
Uys, La Fras	African Institute for Mathematical Sciences

Graduate Students

Alves, Sergio	University of the Witwatersrand
Bruce, Faikah	African Institute for Mathematical Sciences
Chinemerem, Dennis	University of South Africa
Fanucchi, Dario	University of the Witwatersrand
Fareo, Gideon	University of the Witwatersrand
Garth-Davis, Bryan	African Institute for Mathematical Sciences
Kamga Pene, Morgan	University of the Witwatersrand
Korir, Paul Kibet	African Institute for Mathematical Sciences
La Foy, Tanya	University of the Witwatersrand
Lishivha, Lutendo	University of the Witwatersrand
Malan, Yourinde	University of the Witwatersrand
Matebese, Belinda	North-West University
Mistry, Trishna	University of the Witwatersrand
Moepya, Obakeng	University of the Witwatersrand

Mthethwa, Nkosinokuthula	University of the Witwatersrand
Naidoo, Sumita	Durban University of Technology
Ndadza, Tshifhango	University of the Witwatersrand
Newby, Eric	University of the Witwatersrand
Ngabonziza, Prosper	African Institute for Mathematical Sciences
Ngongang Ndjawa, Guy Oliver	African Institute for Mathematical Sciences
Oliphant, Terry-Leigh	University of the Witwatersrand, Johannesburg
Padayachee, Trishanta	University of the Witwatersrand, Johannesburg
Sathinarain, Melisha	University of the Witwatersrand, Johannesburg
Tailor, Asha	University of the Witwatersrand, Johannesburg
Uoane, Tumelo	University of the Witwatersrand, Johannesburg

Industry Representatives

Damelin, Steven	Georgia Southern University, USA
Govender, Indresan	University of Cape Town
Groess, Robert	University of the Witwatersrand, Johannesburg
Morrison, Norman	University of Cape Town
Naidoo, Richard	Durban University of Technology
Thiart, Gerrie	University of Stellenbosch

PROBLEMS AND EXECUTIVE SUMMARIES

A brief description of each problem is given followed by an equation-free Executive Summary of the progress made on the problem.

Detailed Technical Reports are presented in the next section.

POSITRON EMISSION PARTICLE TRACKING

Minerals

Industry Representative

Indresen Govender, Centre for Minerals Research, University of Cape Town

Moderator

Neville Fowkes, University of Western Australia

Student Moderators

James Newling, AIMS; Angus Morrison and Lawrence Bbosa, university of Cape Town

Problem Description

Positron Emission Particle Tracking (PEPT) refers to a procedure for tracing the movement of particles that uses positron generated gamma rays emitted from radioactive materials imbedded in the particles. This procedure can be used to trace the movement of particles in fluid mechanical and granular material flows, and in particular can be used to trace the movement of rocks in the tumbling mills used by the mineral industry. The radioactive source emits back to back gamma rays which are detected when they strike detectors mounted around the tumble mill. Under true pairing conditions the back to back rays form a straight line passing through the particle position and striking the detectors and (using successive emissions) a triangulation procedure can be used to locate the moving source. However, incorrect pairing results if either or both the emitted back to back rays are scattered, or if random pairing occurs and in fact, 70% to 95% of detections are false. The study group was asked to examine effective procedures for locating the true position, velocity and acceleration of the source using the data from the detectors. Data collected from an instrumented laboratory tumble mill was made available.

Executive Summary

Two features of the problem stand out. Firstly, because of the geometry, false lines generally miss the mark by a long shot, so that even if a small proportion of the very large number of such lines are retained they will greatly effect the position calculation. Secondly there is an order of magnitude more true lines than required to follow the particle to the desired accuracy. The detection system is in fact very accurate and under ideal conditions can produce much more precision than required for practical use, at least in the tumbler context; errors arise primarily because of false parings. The implication is that it is essential to aggressively cull the retained lines, but of course it is not easy to identify the true lines and complicated calculations on all possible lines are not computationally desirable. What is required is a computationally simple (possibly crude) true pairing test, followed up by a major culling, then more detailed calculations.

A number of obvious possibilities for culling lines and calculating the expected location of the particle were investigated (in total five) and tested using artificially generated data and real data from laboratory tests. The schemes used made use of position estimates based on past history, a range of distance distribution functions associated with the spacing between candidate lines and successive culling/iteration schemes were used to refine the results. The results obtained will be detailed in the final report.

A METHOD FOR STOCHASTIC ESTIMATION OF COST AND COMPLETION TIME OF A MINING PROJECT

Mining

Industry Representative
Cuthbert Musingwini, School of Mining, University of the Witwatersrand

Moderator
Montaz Ali, University of the Witwatersrand

Student Moderators
Trishna Mistry, University of the Witwatersrand; Dennis Chinemerem, University of South Africa.

Problem Description

When a new mining project is proposed the project proponents are faced with two important tasks, the estimation of completion time and the project cost. It is extremely important that these estimates are as accurate as possible. The problem was submitted by a mining company for which the most important challenge lies in the stochastic nature of the completion times of a number of project components with no known or obvious distribution pattern. The Study Group was asked to devise an algorithm for the stochastic estimation of project completion.

Executive Summary

The Study Group concentrated on completion time. It adopted two methods of estimation of completion time. One was the use of the Monte Carlo technique in the critical path method. The other was the use of a simplifying assumption such as the best distribution in calculating the estimated completion time. Simulations were performed which were encouraging.

WIND TURBINE DESGIN

Renewable energy

Industry Representatives
Richard Naidoo, Durban University of Technology

Moderator
Neville Fowkes, University of Western Australia

Student Moderators
Faikah Bruce AIMS; Tshifhango Ndadza, University of the Witwatersrand; Belinda Matebese, North West University

Problem Description

The brief was to design a 50kW wind turbine for an eco-village in the KwaZulu-Natal coastal region north of Durban with a rated wind speed of 13.5m/sec and where winds vary from 3.5m/sec to 18m/sec. Of particular interest was the axis orientation (horizontal or vertical), the number size and shape of blades and turbine height.

Executive Summary

While detailed engineering design involves issues well beyond those that could be sensibly addressed by the MISG group, we did attempt to set down the aerodynamic design principles for such an undertaking. Our investigations were primarily based on Richard Naidoo's analysis of the problem and results gleaned from David Spera's excellent book 'Wind Turbine Technology' ASME Press NY 1994 and references contained within.

There are two types of wind turbines: vertical axis wind turbines (VAWT) and the conventional horizontal axis turbine (HAWTs), In general terms HAWTs are much more efficient (by a factor of about 7) than VAWTs in steady winds because they utilize lift forces as opposed to drag forces on the airfoils. The VAWTs are more suitable under highly variable wind conditions. It was felt that steady sea breezes off Durban would make the HAWT design more suitable, so we limited our investigations to HAWTs.

The maximum power available from the wind per unit area is ½ ρU_w^3 , where U_w is the wind speed (assumed fixed) and ρ is the density of the air. However, wind turbines can extract no more than 16/27ths of this available power (a theoretical result known as the Betz's limit), so that the maximum possible power output for a turbine with a rotor area A is given by $\frac{16}{27}\left(\frac{1}{2}\rho U_w^3\right)A$. An important and somewhat surprising observation is that

standard design wind turbines with either two or three blades realize this theoretical limit to within about 6% to 10%. The implication is that the primary design features are the rotor area A and the wind speed U_w at the rotor location. Note that the power output varies in proportion to U_w^3 and so is very sensitive to wind speed. As a rule of thumb the wind speed increases by approximately 20% for every additional 10m height H due to wind shear in the ground boundary layer which corresponds to a 34% increase in power output, However tall structures are expensive to build and run. Evidently for a given power output (50kwatts in our case) U_w^3 (H) A is prescribed, so that the optimum (U_w, H) combination is the one that minimizes an appropriate combination of construction and running costs subject to the power output requirement. These results will be detailed in the main report.

The Study Group visited the experimental wind farm, Windhoek, at Darling on the Cape West Coast where there were four wind turbines. This gave the Study Group an idea of the size of the tower and blades.

PRODUCTION OF BIOFUELS USING HYDROLYSIS AND FERMENTATION

Renewable energy

Industry Representative
Sewis Van Zyl, University of Stellenbosch

Moderator
Jean Charpin, University of Limerick, Ireland

Student Moderators
Gideon Fareo, University of the Witwatersrand; La Fras Uys, AIMS

Problem Description

In recent years, ethanol has become an alternative to fossil fuels in the car industry. A production method is currently investigated at the University of Stellenbosch in South Africa. Cellulose particles are mixed with enzymes and yeast in a fermentation tank and after approximately five days of chemical reactions, most of the cellulose has been converted to ethanol. The process is currently being tested in a small tank. However, some problems have emerged concerning the mixing in the tank: the rate of mixing is much faster than the reaction rates of the process. Mixing typically occurs within seconds while the biological reactions take days. The progress of the biological reactions further complicate the modelling by affecting certain fluid properties, for example viscosity. This change in turn alters the mixing conditions within the fermentation tank. The group was asked to study the mixing in a large tank for the extent of the reaction time.

Executive Summary

The study group followed three lines of investigation.

- *Modelling the chemical reactions.*

  The cellulose is mixed in with three types of enzymes and yeast in water. Ethanol is then produced after three chemical reactions and one fermentation step. Initially, the group was provided with a set of equations modelling the variations of concentrations during the reaction. A simpler model was developed and long time behaviour was investigated.

- *Fluid mechanics*
  The reaction takes place in a one thousand cubic metre tank. Cellulose is introduced in the form of small particles assumed to be spherical balls. The reactants are mixed with water using an impeller located close to the bottom of the tank. When the impeller is stopped, the cellulose balls start falling down. This will slow down the reactions and after some time, the impeller has to be started again. The group studied the movement of the cellulose balls and briefly investigated the fluid dynamics in the tank.

- *Optimisation*

  Using the models developed during the week, the group determined an energy efficient use of the impeller. An on-off schedule was determined based on the movements of the cellulose balls. An optimisation argument shows that, under our assumptions, the impeller can be switched on for only a small fraction of time during the five days necessary to complete the reaction and for this well chosen schedule, the cellulose and enzymes are well mixed throughout the reaction volume.

VORTEX LATTICE METHODS FOR HYDROFOILS

Hydrofoils

Industry Representative
Gerrie Thiart, School of Mechanical Engineering, University of Stellenbosch

Moderators
Alistair Fitt, University of Southampton, England; Neville Fowkes, University of Western Australia, Australia

Student Moderators
Eric Newby, University of the Witwatersrand; Prosper Ngabonziza and Eyaya Eneyew, AIMS

Problem Description

A hydrofoil is essentially an airfoil placed under a vessel. Once the vessel is moving the lift force on the airfoil causes the vessel to rise out of the water thereby greatly reducing drag and enabling the vessel to travel faster; high vessel speeds can be achieved economically using hydrofoils. As the vessel speed increases the foil gets closer to the water surface and more waves are generated so drag increases dramatically.

Using the vortex lattice method (see below) Gerrie Thiart has made calculations on the lift and drag on hydrofoils, but his procedure runs into numerical difficulties for foils close to the water surface. In particular certain integrals that need to be evaluated become nearly divergent or exhibit highly oscillatory behaviour in the small foil depth range. He asked the group to determine the nature of the difficulties and if possible to increase the range of applicability of his results to cover this important high speed design range.

Executive Summary

The vortex lattice method is a standard linear technique applied by aerodynamicists to compute the ideal-flow aerodynamic characteristics (lift, induced-drag and centre-of-pressure) of thin wings in a fluid of infinite extent. In the hydrofoil problem the presence of the free surface greatly complicates the situation mathematically and physically. The associated free-surface condition is non-linear and the waves generated at the free surface induce a drag on the foil. For small amplitude waves the surface boundary condition can be linearized so that the vortex lattice technique can be applied

to obtain estimates for the wave drag and lift; this is what Gerrie Thiart has done. The technique represents the solution for the velocity potential in terms of an unknown vortex distribution on the vanishing thin foil surface, and the unknown distribution is determined so that the linearized boundary conditions are satisfied. This leads to a system of linear algebraic equations with coefficients involving integrals that need to be computed numerically. It is these integrals that are troublesome.

Three integrals were identified that could be troublesome for numerical quadrature: a Cauchy Principle Value integral, a hypersingular integral, and a highly oscillatory integral. The hypersingular integral could be avoided by a rearrangement of the calculations, which leads to a less troublesome Cauchy Principle Value integral. In all these cases, however, extreme care is required; normal numerical procedures lead to incorrect results. Spectral methods are recommended for CPV evaluations and asymptotic schemes can be used to handle the highly oscillatory integrals. It is not clear if the numerical schemes Gerrie Thiart used produce incorrect results, but almost assuredly incorrect results would be obtained unless correct measures were taken. It should be noted that for small airfoil depths the theory will fail in a way that cannot be corrected by a more careful analysis. For small enough foil depths non-linear effects cannot be ignored and the very large increase in wave drag is symptomatic of such effects. Estimates for the range of applicability of the linear theory were obtained.

IMAGE RECOGNITION

Internet

Industry Representative
Robert Groess, University of the Witwatersrand

Moderator
Amandine Robin, University of the Witwatersrand

Student Moderator
Paul Korir, AIMS

Problem Description

The Student Group was asked to develop a highly efficient algorithm for recognizing images based exclusively on their image content. This is to be used in an image search engine which would emulate the performance of today's text-based search engines, but using image data instead of text. To this end the following criteria were formalized:

1. The idea is to extract useful features of an image to generate a unique index key.

2. The algorithm is to determine if the entire image contents, or parts thereof, are contained in comparison images (identified from index keys generated from crawling the world wide web).

3. To determine if similar, but not necessarily identical objects are present in an image without requiring human intervention in the recognition process.

4. To do all the above with extreme emphasis on minimizing processing cost and optimizing index key storage efficiency.

Executive Summary

Initial inspiration was drawn from Wang (2003) describing an audio search algorithm that is both extremely fast and very accurate in recognizing audio tracks, even in the presence of noise and general quality degradation. Wang's algorithm breaks an audio track into a frequency space and identifies peak energy points within a given period, which are used to construct a constellation map (or fingerprint). This constellation map is unique to a given song or audio track and is an extremely reliable indicator for identifying professionally recorded music albums. This approach is applicable to images only if a way can be found to extract features enabling unique identification. The possibility of extracting unique features from images is to be investigated further.

Another approach was to divide an image into a predetermined number of sequential "windows" where the maximum values of the pixel intensities in each window form an envelope which is modulated in "intensity-position" space. The envelope modulation is robust against changes in image dimension, up to a limit where an image would be so small that the "frequency" in "intensity-position" space would be lower than the size of the envelope features. This concept was explored even further to develop a unique image "fingerprint" based on the values of the image pixel intensity peaks. These peaks form an n-dimensional vector which can be compared with other vectors using vector algebra. This gives a mathematical way of relating how similar two images are by the distance between their n-dimensional fingerprint vectors. Future work on refining this method should include mathematical optimization of the window sizes and dimensionality of the fingerprint keys used.

IDENTIFICATION OF NERVES IN UPPER BODY ULTRASOUNDS

Medical

Industry Representative
Steven Damelin, Georgia Southern University, USA; Michael Sears, University of the Witwatersrand

Moderator
Michael Mitcheley, University of the Witwatersrand

Student Moderator
Dario Fanucchi, University of the Witwatersrand

Problem Description

Ultrasound has been shown to be an aid to peripheral nerve block. Amongst its advantages are higher success rates, shorter onset times, a decreased incidence of vascular puncture and a faster learning curve to master the techniques of regional anesthesia. One of the skills necessary to conduct ultrasound guided nerve blocks is the ability to recognize the nerves, vessels, muscles and bones in sagittal and axial cross sections. In fit healthy patients, these structures are reasonably easy to recognize. But in obese patients, the extra adipose tissue attenuates the ultrasound beam. In this situation it can be very difficult to identify the nerves.

The Study Group was asked to develop techniques in image processing for the identification of nerve fibres within ultrasound images.

Executive summary

A number of possible methods for accomplishing the identification of nerve fibres were explored. Preliminary results indicated that texture-based segmentation of the image was likely to reveal nerve clusters, provided a medical professional supervised the process. Another possible approach explored was that of improving the image quality through image restriction across multiple frames. An appropriate model of the possible distortions would be needed.

Technical Reports

The following technical reports have been received.

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Student Moderators
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Neville Fowkes, University of Western Australia

Student Moderators
Faikah Bruce AIMS; Tshifhango Ndadza, University of the Witwatersrand; Belinda Matebese, North West University

Problem Description
The brief was to design a 50kW wind turbine for an eco-village in the KwaZulu-Natal coastal region north of Durban with a rated wind speed of 13.5m/sec and where winds vary from 3.5m/sec to 18m/sec. Of particular interest was the axis orientation (horizontal or vertical), the number size and shape of blades and turbine height.

Executive Summary
While detailed engineering design involves issues well beyond those that could be sensibly addressed by the MISG group, we did attempt to set down the aerodynamic design principles for such an undertaking. Our investigations were primarily based on Richard Naidoo's analysis of the problem and results gleaned from David Spera's excellent book 'Wind Turbine Technology' ASME Press NY 1994 and references contained within.

There are two types of wind turbines: vertical axis wind turbines (VAWT) and the conventional horizontal axis turbine (HAWTs), In general terms HAWTs are much more efficient (by a factor of about 7) than VAWTs in steady winds because they utilize lift forces as opposed to drag forces on the airfoils. The VAWTs are more suitable under highly variable wind conditions. It was felt that steady sea breezes off Durban would make the HAWT design more suitable, so we limited our investigations to HAWTs.

The maximum power available from the wind per unit area is ½ ρU_w^3 , where U_w is the wind speed (assumed fixed) and ρ is the density of the air. However, wind turbines can extract no more than 16/27ths of this available power (a theoretical result known as the Betz's limit), so that the maximum possible power output for a turbine with a rotor area A is given by $\frac{16}{27}\left(\frac{1}{2}\rho U_w^3\right)A$. An important and somewhat surprising observation is that

standard design wind turbines with either two or three blades realize this theoretical limit to within about 6% to 10%. The implication is that the primary design features are the rotor area A and the wind speed U_w at the rotor location. Note that the power output varies in proportion to U_w^3 and so is very sensitive to wind speed. As a rule of thumb the wind speed increases by approximately 20% for every additional 10m height H due to wind shear in the ground boundary layer which corresponds to a 34% increase in power output, However tall structures are expensive to build and run. Evidently for a given power output (50kwatts in our case) U_w^3 (H) A is prescribed, so that the optimum (U_w, H) combination is the one that minimizes an appropriate combination of construction and running costs subject to the power output requirement. These results will be detailed in the main report.

The Study Group visited the experimental wind farm, Windhoek, at Darling on the Cape West Coast where there were four wind turbines. This gave the Study Group an idea of the size of the tower and blades.

PRODUCTION OF BIOFUELS USING HYDROLYSIS AND FERMENTATION

Renewable energy

Industry Representative
Sewis Van Zyl, University of Stellenbosch

Moderator
Jean Charpin, University of Limerick, Ireland

Student Moderators
Gideon Fareo, University of the Witwatersrand; La Fras Uys, AIMS

Problem Description

In recent years, ethanol has become an alternative to fossil fuels in the car industry. A production method is currently investigated at the University of Stellenbosch in South Africa. Cellulose particles are mixed with enzymes and yeast in a fermentation tank and after approximately five days of chemical reactions, most of the cellulose has been converted to ethanol. The process is currently being tested in a small tank. However, some problems have emerged concerning the mixing in the tank: the rate of mixing is much faster than the reaction rates of the process. Mixing typically occurs within seconds while the biological reactions take days. The progress of the biological reactions further complicate the modelling by affecting certain fluid properties, for example viscosity. This change in turn alters the mixing conditions within the fermentation tank. The group was asked to study the mixing in a large tank for the extent of the reaction time.

Executive Summary

The study group followed three lines of investigation.

- *Modelling the chemical reactions.*

  The cellulose is mixed in with three types of enzymes and yeast in water. Ethanol is then produced after three chemical reactions and one fermentation step. Initially, the group was provided with a set of equations modelling the variations of concentrations during the reaction. A simpler model was developed and long time behaviour was investigated.

- *Fluid mechanics*
  The reaction takes place in a one thousand cubic metre tank. Cellulose is introduced in the form of small particles assumed to be spherical balls. The reactants are mixed with water using an impeller located close to the bottom of the tank. When the impeller is stopped, the cellulose balls start falling down. This will slow down the reactions and after some time, the impeller has to be started again. The group studied the movement of the cellulose balls and briefly investigated the fluid dynamics in the tank.

- *Optimisation*

  Using the models developed during the week, the group determined an energy efficient use of the impeller. An on-off schedule was determined based on the movements of the cellulose balls. An optimisation argument shows that, under our assumptions, the impeller can be switched on for only a small fraction of time during the five days necessary to complete the reaction and for this well chosen schedule, the cellulose and enzymes are well mixed throughout the reaction volume.

VORTEX LATTICE METHODS FOR HYDROFOILS

Hydrofoils

Industry Representative
Gerrie Thiart, School of Mechanical Engineering, University of Stellenbosch

Moderators
Alistair Fitt, University of Southampton, England; Neville Fowkes, University of Western Australia, Australia

Student Moderators
Eric Newby, University of the Witwatersrand; Prosper Ngabonziza and Eyaya Eneyew, AIMS

Problem Description

A hydrofoil is essentially an airfoil placed under a vessel. Once the vessel is moving the lift force on the airfoil causes the vessel to rise out of the water thereby greatly reducing drag and enabling the vessel to travel faster; high vessel speeds can be achieved economically using hydrofoils. As the vessel speed increases the foil gets closer to the water surface and more waves are generated so drag increases dramatically.

Using the vortex lattice method (see below) Gerrie Thiart has made calculations on the lift and drag on hydrofoils, but his procedure runs into numerical difficulties for foils close to the water surface. In particular certain integrals that need to be evaluated become nearly divergent or exhibit highly oscillatory behaviour in the small foil depth range. He asked the group to determine the nature of the difficulties and if possible to increase the range of applicability of his results to cover this important high speed design range.

Executive Summary

The vortex lattice method is a standard linear technique applied by aerodynamicists to compute the ideal-flow aerodynamic characteristics (lift, induced-drag and centre-of-pressure) of thin wings in a fluid of infinite extent. In the hydrofoil problem the presence of the free surface greatly complicates the situation mathematically and physically. The associated free-surface condition is non-linear and the waves generated at the free surface induce a drag on the foil. For small amplitude waves the surface boundary condition can be linearized so that the vortex lattice technique can be applied

to obtain estimates for the wave drag and lift; this is what Gerrie Thiart has done. The technique represents the solution for the velocity potential in terms of an unknown vortex distribution on the vanishing thin foil surface, and the unknown distribution is determined so that the linearized boundary conditions are satisfied. This leads to a system of linear algebraic equations with coefficients involving integrals that need to be computed numerically. It is these integrals that are troublesome.

Three integrals were identified that could be troublesome for numerical quadrature: a Cauchy Principle Value integral, a hypersingular integral, and a highly oscillatory integral. The hypersingular integral could be avoided by a rearrangement of the calculations, which leads to a less troublesome Cauchy Principle Value integral. In all these cases, however, extreme care is required; normal numerical procedures lead to incorrect results. Spectral methods are recommended for CPV evaluations and asymptotic schemes can be used to handle the highly oscillatory integrals. It is not clear if the numerical schemes Gerrie Thiart used produce incorrect results, but almost assuredly incorrect results would be obtained unless correct measures were taken. It should be noted that for small airfoil depths the theory will fail in a way that cannot be corrected by a more careful analysis. For small enough foil depths non-linear effects cannot be ignored and the very large increase in wave drag is symptomatic of such effects. Estimates for the range of applicability of the linear theory were obtained.

IMAGE RECOGNITION

Internet

Industry Representative
Robert Groess, University of the Witwatersrand

Moderator
Amandine Robin, University of the Witwatersrand

Student Moderator
Paul Korir, AIMS

Problem Description

The Student Group was asked to develop a highly efficient algorithm for recognizing images based exclusively on their image content. This is to be used in an image search engine which would emulate the performance of today's text-based search engines, but using image data instead of text. To this end the following criteria were formalized:

1. The idea is to extract useful features of an image to generate a unique index key.

2. The algorithm is to determine if the entire image contents, or parts thereof, are contained in comparison images (identified from index keys generated from crawling the world wide web).

3. To determine if similar, but not necessarily identical objects are present in an image without requiring human intervention in the recognition process.

4. To do all the above with extreme emphasis on minimizing processing cost and optimizing index key storage efficiency.

Executive Summary

Initial inspiration was drawn from Wang (2003) describing an audio search algorithm that is both extremely fast and very accurate in recognizing audio tracks, even in the presence of noise and general quality degradation. Wang's algorithm breaks an audio track into a frequency space and identifies peak energy points within a given period, which are used to construct a constellation map (or fingerprint). This constellation map is unique to a given song or audio track and is an extremely reliable indicator for identifying professionally recorded music albums. This approach is applicable to images only if a way can be found to extract features enabling unique identification. The possibility of extracting unique features from images is to be investigated further.

Another approach was to divide an image into a predetermined number of sequential "windows" where the maximum values of the pixel intensities in each window form an envelope which is modulated in "intensity-position" space. The envelope modulation is robust against changes in image dimension, up to a limit where an image would be so small that the "frequency" in "intensity-position" space would be lower than the size of the envelope features. This concept was explored even further to develop a unique image "fingerprint" based on the values of the image pixel intensity peaks. These peaks form an *n*-dimensional vector which can be compared with other vectors using vector algebra. This gives a mathematical way of relating how similar two images are by the distance between their *n*-dimensional fingerprint vectors. Future work on refining this method should include mathematical optimization of the window sizes and dimensionality of the fingerprint keys used.

IDENTIFICATION OF NERVES IN UPPER BODY ULTRASOUNDS

Medical

Industry Representative
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Moderator
Michael Mitcheley, University of the Witwatersrand

Student Moderator
Dario Fanucchi, University of the Witwatersrand

Problem Description
Ultrasound has been shown to be an aid to peripheral nerve block. Amongst its advantages are higher success rates, shorter onset times, a decreased incidence of vascular puncture and a faster learning curve to master the techniques of regional anesthesia. One of the skills necessary to conduct ultrasound guided nerve blocks is the ability to recognize the nerves, vessels, muscles and bones in sagittal and axial cross sections. In fit healthy patients, these structures are reasonably easy to recognize. But in obese patients, the extra adipose tissue attenuates the ultrasound beam. In this situation it can be very difficult to identify the nerves.

The Study Group was asked to develop techniques in image processing for the identification of nerve fibres within ultrasound images.

Executive summary
A number of possible methods for accomplishing the identification of nerve fibres were explored. Preliminary results indicated that texture-based segmentation of the image was likely to reveal nerve clusters, provided a medical professional supervised the process. Another possible approach explored was that of improving the image quality through image restriction across multiple frames. An appropriate model of the possible distortions would be needed.

Technical Reports

The following technical reports have been received.