Lane-emden equation of the second kind

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Overviews

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- Introduction
- Derivation of Lane-Emden Equation
- Numerical Solutions
- Comparison with Frank Kamenetskii
- Conclusion
- Questions

Introduction

- The Lane-Emden value equation of the second kind

$$y'' + \frac{k}{x}y' + \exp y = 0 \tag{1}$$

where $k = 1$,subject to the boundary condition $y'(0) = 0$ and $y(1) = 0$

Introduction

- The Lane-Emden value equation of the second kind

$$y'' + \frac{k}{x}y' + \exp y = 0 \qquad (1)$$

  where $k = 1$,subject to the boundary condition $y'(0) = 0$ and $y(1) = 0$
- Consider the Poisson-Boltzmann equations which models the thermal explosion in cylindar
- Reduce the Poisson-Boltzmann equations to Lane-Emden equation

Derivation of Lane-Emden Equation

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Poisson-Boltzmann equation

$$\theta'' + \frac{1}{z}\theta' + \delta \exp\theta = 0, \qquad (2)$$

with the boundary

$$\theta'(0) = 0 \text{ and } \theta(1) = 0 \qquad (3)$$

Derivation of Lane-Emden Equation

Poisson-Boltzmann equation

$$\theta'' + \frac{1}{z}\theta' + \delta \exp \theta = 0, \tag{2}$$

with the boundary

$$\theta'(0) = 0 \text{ and } \theta(1) = 0 \tag{3}$$

Change the variables using this transformations

$$y = \theta - \theta_0, x = z[\delta \exp \theta_0]^{\frac{1}{2}} \tag{4}$$

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get the Lane-Emden equation of the second kind

$$y^{''} + \frac{1}{x}y^{'} + \exp y = 0 \tag{5}$$

subject to the boundary condition

$$y^{'}(0) = 0, y(R) = -\theta_0 \tag{6}$$

where $R = [\delta e^{\theta_0}]^{\frac{1}{2}}$

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- Finite Difference Method to approximate the ODE(Lane-Emden)

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- Singularity case $x = 0$, By Shampine we know that

$$\frac{1}{x}y' \approx y'' \tag{7}$$

eqn(5) become

$$y'' + \exp y = 0 \tag{8}$$

and after the applying finite difference eqn(8) we get

$$y_{i-1} - 2y_i + y_{i+1} = -\frac{1}{2}h^2 \exp y_i \tag{9}$$

- Non-Singular cases $x \neq 0$ we get

$$\alpha_i y_{i-1} - y_i + \beta_i y_{i+1} = -\frac{1}{2}h^2 \exp y_i \tag{10}$$

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- Determine the linear Equations for all $i's$ to Ay=b and use Jacobs Method to calculate y values iterative

Comparison with Frank Kamenetskii

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- Critical point of explosion is where $\delta = 2$ as proven analytical by Frank Kamenetskii

Conclusion

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- $\delta = 2$ indicate that the explosion has occured

Questions

THANK YOU !!